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Conformal invariance in the long-range Ising model

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Abstract

We consider the question of conformal invariance of the long-range Ising model at the critical point. The continuum description is given in terms of a nonlocal field theory, and the absence of a stress tensor invalidates all of the standard arguments for the enhancement of scale invariance to conformal invariance. We however show that several correlation functions, computed to second order in the epsilon expansion, are nontrivially consistent with conformal invariance. We proceed to give a proof of conformal invariance to all orders in the epsilon expansion, based on the description of the long-range Ising model as a defect theory in an auxiliary higher-dimensional space. A detailed review of conformal invariance in the d-dimensional short-range Ising model is also included and may be of independent interest.

1. Introduction

We will be studying the long-range Ising (LRI) model—a close cousin of the usual, short-range, Ising model (SRI). While in the SRI only nearest-neighbor spins interact, the LRI energy involves interactions at arbitrary distances with a powerlaw potential:

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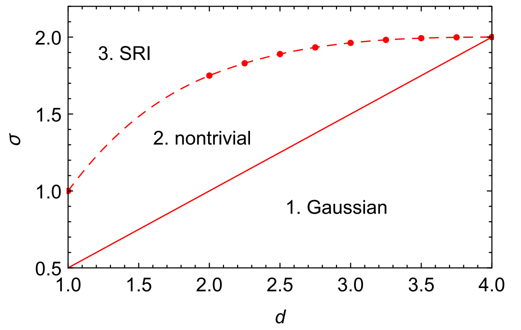

Fig. 1. The three phases for the LRI critical point. The boundaries are $\sigma = d/2$ (straight solid) and $\sigma = 2 - \eta_{\rm SRI}(d)$ (curved dashed, interpolated using the known exact $\eta_{\rm SRI}(d)$ for $d = 1, 2, 4$ and numerical values from the ϵ-expansion [1] and the conformal bootstrap [2] for a few intermediate d). Here we will be working near the boundary $\sigma = d/2$.

$$H_{\rm LRI} = -J \sum_{i,j} s_i s_j / r_{ij}^{d+\sigma} \,, \tag{1.1}$$

where d is the space dimensionality, and $\sigma > 0$ is a free parameter. The sum is over all pairs of lattice sites and r_{ij} is the distance between them. We are considering the ferromagnetic interaction case $J > 0$.

Just like the SRI, the LRI has a second-order phase transition at a critical temperature $T = T_c$. The critical theory is universal, i.e. independent of the short-distance details such as the choice of the lattice. It has however an interesting dependence on σ (see Fig. 1):

1. For $\sigma < d/2$, the critical point is a Gaussian theory described by the nonlocal action involving the "fractional Laplacian" operator $\mathcal{L}_\sigma \equiv (-\partial^2)^{\sigma/2}$:
$$S_0 = \int \mathrm{d}^d x\, \mathrm{d}^d y\, \phi(x)\phi(y)/|x-y|^{d+\sigma} \propto \int \mathrm{d}^d x\, \phi\, \mathcal{L}_\sigma \phi \,. \tag{1.2}$$
The field ϕ represents the spin density, and its scaling dimension is read off as
$$\Delta_\phi = (d-\sigma)/2 \,. \tag{1.3}$$
Composite operators can be built out of ϕ by differentiations and taking normal-ordered products. Since the theory is Gaussian, there are no anomalous dimensions.
2. For $d/2 < \sigma < \sigma_*$, the critical point is a nontrivial, non-Gaussian, theory, whose field-theoretic description can be obtained by perturbing the nonlocal action S_0 by a local quartic interaction $\int \mathrm{d}^d x\, \phi^4(x)$. In the range $\sigma > d/2$ this interaction is relevant and generates a renormalization group (RG) flow, reaching a fixed point in the IR. This IR fixed point is believed to be in the same universality class as the critical point of the LRI lattice model. Interestingly, as it will be explained below, the dimension of ϕ at the fixed point is still given by the same formula (1.3). However, composite operators do get nontrivial anomalous dimensions.
3. Finally, for $\sigma > \sigma_*$ the potential is so strongly peaked at short distances that the LRI critical point is identical with the SRI critical point, i.e. is independent of σ. The value of σ_* is determined so that the dimension of ϕ is continuous across this transition:
$$\sigma_* = d - 2\Delta_\phi^{\rm SRI} \equiv 2 - \eta_{\rm SRI} \,, \tag{1.4}$$
using the usual definition of the η critical exponent.