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Quasilocal conservation laws from semicyclic irreducible representations of $U_q(\mathfrak{sl}_2)$ in XXZ spin-1/2 chains

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Abstract

We construct quasilocal conserved charges in the gapless ($|\Delta| \leq 1$) regime of the Heisenberg XXZ spin-1/2 chain, using semicyclic irreducible representations of $U_q(\mathfrak{sl}_2)$. These representations are characterized by a periodic action of ladder operators, which act as generators of the aforementioned algebra. Unlike previously constructed conserved charges, the new ones do not preserve magnetization, i.e. they do not possess the $U(1)$ symmetry of the Hamiltonian. The possibility of application in relaxation dynamics resulting from $U(1)$-breaking quantum quenches is discussed.

1. Introduction

In this paper we shall consider the anisotropic Heisenberg XXZ spin-1/2 chain with periodic boundary conditions, from the point of view of theory of integrability. Heisenberg spin chains, providing successful theoretical description of magnetism-related phenomena in spin-

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chain materials [1], can be studied analytically using the so called *algebraic Bethe ansatz* method [2,3], resulting in an infinite family of conservation laws. These can then be applied in numerous ways to treat various aspects of equilibrium and nonequilibrium statistical physics of the model in consideration. An example of application of these results out-of-equilibrium is a rigorous derivation and evaluation [4–7] of Mazur–Suzuki lower bound [8,9] for spin transport [10]. The result of this evaluation in linear response theory implies strict ballistic property of high temperature spin transport in the thermodynamic limit in the $|\Delta| < 1$ regime of the anisotropic Heisenberg spin-$1/2$ Hamiltonian. This issue has been controversial in view of the fact that the *thermodynamic Bethe ansatz* approach to quantum spin transport allowed for different, mutually inconsistent results [11,12]. Another, closely related application of (quasi)local conserved operators is in relaxation dynamics that follows quantum quenches of integrable systems, where the precise formulation of the so-called *generalized Gibbs ensemble* is currently under intense investigation [13–15].

The success of the algebraic Bethe ansatz method, in case of Heisenberg spin chains relies heavily on the existence of quantum group $U_q(\mathfrak{sl}_2)$ and its universal R-matrix, satisfying the so called *quantum Yang–Baxter equation* [16]. Using the fundamental, two-dimensional representations of these objects, Faddeev and his Leningrad school [2] developed a general technique which generates the Heisenberg Hamiltonian, together with the full family of local conserved charges in involution, via the logarithmic derivatives of the quantum transfer matrix. However, in a recent progress [6,7,17], other highest weight irreducible representations at the root-of-unity values of q, which densely populate the entire critical interval $-1 < \Delta < 1$, have been implemented to construct quasi-local conserved quantities relevant for quantum spin transport and quantum relaxation [18]. In the present paper, these constructions are generalized and extended, using *semicyclic* irreducible representations [19,20]. Here the highest and the lowest weight vectors are coupled by the periodic action of $U_q(\mathfrak{sl}_2)$-generators.

In the second section we briefly review the model and the structure of cyclic and semicyclic irreducible representations of $U_q(\mathfrak{sl}_2)$, as given (up to module isomorphism) in Refs. [19] and [20]. Also, the problem of periodicity of the generator action is reviewed: the fundamental commutation relations described in [2], implying the conservation and involution of transfer-matrix related quantities, fail to hold in case of irreducible even-dimensional and certain odd-dimensional (semi)cyclic representations [21]. Since the anisotropy parameter is linked to the parameter q – the parameter of deformation of the quantum group and thus to the dimension of a representation, this imposes a restriction on the values of anisotropy parameter, for which this construction is valid.

In the third section we construct new quasilocal conserved quantities (31), from valid odd-dimensional irreducible representations, using the formal procedure described in [6]. Quasilocality follows from the argument stated in the latter paper. As a direct consequence of periodicity of generator actions, the total magnetization in z direction is not conserved by the newly constructed conserved operators. This is compatible with abundant degeneracies found in the spectrum of the *XXZ* model at the root of unity anisotropies as a consequence of the loop algebra symmetry [22,23].

Finally, we propose in section 4 some potentially interesting applications of new quasilocal conserved operators for computing generalized Drude weights and quantum relaxation dynamics resulting from $U(1)$-symmetry breaking quantum quenches in the regime of linear response.