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Conformal correlation functions in the Brownian loop soup

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Abstract

We define and study a set of operators that compute statistical properties of the Brownian loop soup, a conformally invariant gas of random Brownian loops (Brownian paths constrained to begin and end at the same point) in two dimensions. We prove that the correlation functions of these operators have many of the properties of conformal primaries in a conformal field theory, and compute their conformal dimension. The dimensions are real and positive, but have the novel feature that they vary continuously as a periodic function of a real parameter. We comment on the relation of the Brownian loop soup to the free field, and use this relation to establish that the central charge of the loop soup is twice its intensity.

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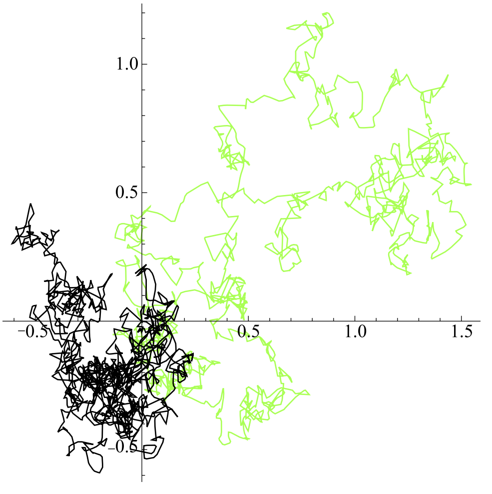

Fig. 1. Two Brownian loops, each of time length $t = 1$.

1. Introduction

1.1. The Brownian loop soup

Take a handful of loops of various sizes and sprinkle them onto a flat surface. The position where each loop lands is uniformly random, independent of any loops already in place. Each loop is Brownian – a Brownian motion constrained to begin and end at the same "root" point, but otherwise with no restriction – and characterized by a "time" length t that is linearly related to its average area (*cf.* Fig. 1). The distribution in t is $\sim dt/t^2$, so that there are many more small loops than large, and is chosen to ensure invariance under scale transformations. The overall density of loops is characterized by a single parameter: the "intensity" $\lambda > 0$. This random ensemble of loops (Fig. 2) is called the Brownian Loop Soup (BLS) and was introduced in [1].

More precisely, the BLS is a Poissonian random collection of loops in a planar domain D with intensity measure $\lambda\mu_D^{loop}$, where $\lambda > 0$ is a constant and μ_D^{loop} is the restriction to D of the *Brownian loop measure*

$$\mu^{loop} = \int_{\mathbb{C}} \int_0^\infty \frac{1}{2\pi t^2}\, \mu_{z,t}^{br}\, dt\, d\mathbf{A}(z), \tag{1.1}$$

where $\mathbf{A}$ denotes area and $\mu_{z,t}^{br}$ is the complex Brownian bridge measure with starting point z and duration t. We note that the Brownian loop measure should be interpreted as a measure on "unrooted" loops, that is, loops without a specified "root" point. (Formally, unrooted loops are equivalence classes of rooted loops – the interested reader is referred to [1] for the details.) For ease of notation, the μ^{loop}-measure of a set $\{\ldots\}$ will be denoted $\mu^{loop}(\ldots) \equiv \mu^{loop}(\{\ldots\})$.

The BLS turns out to be not just scale invariant, but fully conformally invariant. For sufficiently low intensities λ, the intersecting loops form clusters whose outer boundaries are distributed like Conformal Loop Ensembles (CLEs) [2]. CLEs are the unique ensembles of planar, non-crossing and non-self-crossing loops satisfying a natural conformal restriction property that is conjecturally satisfied by the continuum scaling limits of interfaces in two-dimensional models