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Breaking classical Lie groups to finite subgroups – an automated approach

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Abstract

The decomposition of representations of compact classical Lie groups into representations of finite subgroups is discussed. A MATHEMATICA package is presented that can be used to compute these branching rules using the Weyl character formula. For some low order finite groups including A_4 and $\Delta(27)$ general analytical formulas are presented for the branching rules of arbitrary representations of their smallest Lie super-groups.

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1. Introduction

The Standard Model of particle physics (SM) provides a highly accurate description of Nature. However, there are still many questions to be answered. Amongst others, the so-called flavour puzzle remains to be solved, i.e. a satisfactory explanation of the family structure with its masses and mixing is still lacking. One possible avenue towards a solution is provided by settings with non-abelian discrete flavour or horizontal symmetries, which, somewhat complementary to grand unified symmetries, relate to the different families of the SM. Many models using such symmetries have been built, especially for the lepton sector, where until some time ago the so-called tri-bi-maximal mixing pattern seemed to be in good agreement with observation (for reviews of such models see, for example, [1–6]).

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Even if one is able to find an explanation for the flavour structure of the SM in terms of finite non-abelian symmetries, this is, of course, only the first step. One would also like to explain the origin of these symmetries. This problem is amplified because global symmetries are believed to be broken by gravitational effects [7,8]. One possible consistent origin of these symmetries is provided by string theory [9–12]. However, although in principle highly predictive, string theory has not yet been entirely successful in obtaining unambiguous predictions that can be compared with present-day experimental data. Another, more bottom-up possibility to obtain discrete symmetries that are protected from violation by gravitational effects is by breaking a (non-anomalous) continuous gauge group like $\mathrm{SU}(N)$ [13]. Symmetries of this kind are known as discrete gauge symmetries.

The aim of this work is to aid the construction of models where a finite symmetry emerges from the spontaneous breaking of a continuous symmetry, i.e. from breaking Lie groups to finite subgroups. This is only possible if there is an irreducible representation of the Lie group that contains a trivial singlet of the subgroup. A vacuum expectation value of this component may then break the Lie group to the desired subgroup. Therefore, a procedure is outlined and implemented in the MATHEMATICA [14] package `DecomposeLGReps` allowing to decompose the Lie group's representations into irreducible representations of the subgroup.

The decomposition of a representation of a finite group into representations of a subgroup proceeds via the scalar product of characters. The same technique can, in principle, be used for Lie groups as long as their dimension is finite. However, whereas character tables of finite groups contain all information needed to go through this procedure, it is clearly impossible to compile all the necessary information for Lie groups; the table had to be infinitely large.

Hence, previous studies used different methods to compute the branching rules. A first possibility is using the fact that each Lie group representation can be obtained from the tensor product of fundamental representations[1] as in [15].[2] A second option is working with explicit realisations of the Lie group representations [16]. Both approaches cannot be easily generalised to larger Lie group representations or larger rank Lie groups. In another approach that also highlights the connection between spontaneous and explicit symmetry breaking, Merle and Zwicky [17] used an algorithm based on group invariants and provided a MATHEMATICA package implementing the algorithm for SU(3). Again this is not easily generalised, and the method relies on somewhat advanced notions of invariant theory. Similar considerations also lead to the so-called generating functions for subgroup scalars compiled in [18], which, however, mainly focuses on Lie subgroups of Lie groups. To overcome these limitations, the present work uses the standard technique of the character scalar product and computes the characters on the fly with MATHEMATICA using what is called the Weyl character formula [19] (for more modern treatments see, for example, [20,21]). This, in principle, allows the computation of branching rules for all compact Lie groups and arbitrary finite subgroups thereof. The corresponding MATHEMATICA package `DecomposeLGReps` implementing the formulas for the classical Lie groups $\mathrm{U}(N)$, $\mathrm{SU}(N)$, $\mathrm{SO}(N)$ and $\mathrm{USp}(2N)$ can be found online.[3]

Of course, there remain some general issues with this type of model building. For example, the VEV of the singlet component of the Lie group representation under consideration may be left invariant by a larger number of transformations than the desired subgroup, i.e. the subgroup

[1] So-called spinor representation of $\mathrm{SO}(N)$ are an exception, see Section 2 below.

[2] The title of the present work is an allusion to the title of this reference: 'Spontaneous breaking of SU(3) to finite family symmetries – a pedestrian's approach' by Luhn.

[3] http://einrichtungen.ph.tum.de/T30e/codes/DecomposeLGReps.

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