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Nuclear Physics B 898 (2015) 30–52

NUCLEAR PHYSICS B

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# Boundary conditions and the generalized metric formulation of the double sigma model

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Received 29 April 2015; received in revised form 17 June 2015; accepted 19 June 2015

Available online 30 June 2015

Editor: Stephan Stieberger

**Abstract**

Double sigma model with strong constraints is equivalent to the ordinary sigma model by imposing a self-duality relation. The gauge symmetries are the diffeomorphism and one-form gauge transformation with the strong constraints. We consider boundary conditions in the double sigma model from three ways. The first way is to modify the Dirichlet and Neumann boundary conditions with a fully $O(D, D)$ description from double gauge fields. We perform the one-loop $\beta$ function for the constant background fields to find low-energy effective theory without using the strong constraints. The low-energy theory can also have $O(D, D)$ invariance as the double sigma model. The second way is to construct different boundary conditions from the projectors. The third way is to combine the antisymmetric background field with field strength to redefine an $O(D, D)$ generalized metric. We use this generalized metric to reconstruct a consistent double sigma model with the classical and quantum equivalence.



## 1. Introduction

Duality shows the nontrivial equivalence between two theories. It gives us a hope to unify all known theories. It is one of the important problems in the M-theory. For the ten dimensional theories, we have the T- and S-duality. The T-duality is an equivalence between different radii. We exchange the momentum and winding modes in closed string theory and the Dirichlet and

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http://dx.doi.org/10.1016/j.nuclphysb.2015.06.019

Neumann boundary conditions in open string theory. The T-duality suffers from the T-fold problem in the closed string field theory [1,2]. Even for simple constant flux situation, we still find non-single valued fields because of non-isometry. The T-duality is not a well-defined transition function as gauge transformation or diffeomorphism. The S-duality is an equivalence between strong and weak coupling constants. Therefore, the S-duality is a non-perturbative duality so we cannot use perturbation with the coupling constant parameter. Invalidity of perturbation gives rise to a trouble. As a familiar example, it is the electric–magnetic duality of the abelian gauge theory. In the case of the non-abelian groups, the electric–magnetic duality is an open issue. In eleven dimensions, we combine the T- and S-duality to form the U-duality. The U-duality is expected to be a symmetry of the eleven dimensional supergravity.

The method of solving the T-fold problem is to extend from local to global geometry. So far many low-energy effective theories [3–10] are defined on local geometry. The brane theory with global geometry is the generalized Dirac–Born–Infeld (DBI) theory [11,12]. A non-commutative geometry of this theory at the semi-classical level (constant field strength) is governed by the generalized metric, which is an important element to combine tangent with cotangent bundle. This gives us a new perspective to construct the low-energy effective theory or extend understanding on the T-fold. The low-energy effective theory has a corresponding sigma model [11–14] from the new generalized metric. If we combine vector with one-form, a double geometry appears in their theories. This new geometry possibly be a good description to describe string theory [15,16]. They double coordinates (normal and dual coordinates) to embed the T-duality rule in the $O(D, D)$ structure for the closed string theory [17–28]. This extension gives the Courant bracket, which shows a way to solve the T-fold problem [29,30]. Its extension helps us to define exotic brane. The source of exotic brane is non-geometric flux ($Q$- and $R$-flux). One example is the $5^2_2$-brane theory [31]. This brane theory comes from the Neveu–Schwarz five-brane (NS5-brane) by performing two times T-duality. The double geometry suffers from constraints. The relaxing constraints [32] is a hard problem due to the generalized Lie derivative is not a closed algebra without applying constraints in the double geometry. Recent reviews of double geometry are in [33–35]. For the same understanding of the U-duality as the T-duality, we need to extend this double geometry to the exceptional field theory or exceptional generalized geometry [36,37].

Double geometry of open string is proposed from [38]. They use the similar ways with the closed string theory and suggest that the projectors should satisfy the boundary conditions. The gauge transformation and properties [39,40] of a theory can be understood from the generalized geometry [41,42]. The extension of the gauge transformation from the generalized geometry to double geometry of the ten dimensional supergravity is governed by the $F$-bracket [43]. The strong constraints (removing the dependence of the dual coordinates) of the $F$-bracket has an exact one-form difference from the Courant bracket. A double sigma model with open string is found from this gauge transformation with classical equivalence and quantum equivalence at one-loop level [44]. Quantum fluctuation of string theory also gives a higher derivative gravity theory at low-energy level [45]. One-loop $\beta$ function of a double sigma model for the closed string with the dilaton gives the consistent low-energy effective action [46,47]. The conditions of the quantum conformal and Lorentz invariance are also shown in [48]. The most interesting case of the one-loop quantum fluctuation is to simultaneously consider the fluctuation of the ordinary and dual coordinates, which gives us the correct equation of motion for the generalized metric [49]. This calculation exactly shows a low-energy effective action of the generalized metric formulation [27]. The covariant version of the double sigma model is constructed in [50].