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Leading logarithms for the nucleon mass

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Abstract

Within the heavy baryon chiral perturbation theory approach, we have studied the leading logarithm behavior of the nucleon mass up to four-loop order exactly and we present some results up to six-loop order as well as an all-order conjecture. The same methods allow to calculate the main logarithm multiplying the terms with fractional powers of the quark mass. We calculate thus the coefficients of $m^{2n+1}\log^{(n-1)}(\mu^2/m^2)$ and $m^{2n+2}\log^{n}(\mu^2/m^2)$, with m the lowest-order pion mass. A side result is the leading divergence for a general heavy baryon loop integral.

1. Introduction

The calculation of high order terms in low-energy effective field theories (EFTs) is a difficult task. Nowadays, most interesting observables have been calculated at the second order of the expansion, and the difficulty of these calculations shows little hope for any further expansion. The main problem which restricts the potential of EFTs is their non-renormalizability. The non-renormalizability does not bring any problem, in principle, for the calculation by means of counting schemes for EFTs, first introduced in [1]. However, the rapidly increasing number of low-energy coupling constants (LECs), makes very high order applications practically of little use in general.

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Nevertheless, there are contributions of the higher order terms which are free from higher order LECs. In particular this is true for the leading logarithmical (LLog) contributions. LLogs are not in general dominant for a generic observable. However, for some observables the LLog contribution is dominant. Examples of such observables are the generalized parton distributions at small-x [2,3] and certain $\pi\pi$ scattering lengths [4]. In addition, the LLog terms are of great theoretical interest because they allow us to judge the behavior of a whole series of corrections in EFTs. We therefore consider the calculation of LLog terms in EFTs as an interesting and useful task.

In renormalizable field theories the LLog terms can be calculated to all orders using the renormalization group (RG) and (simple) one-loop calculations of the beta functions. In EFTs, as e.g. chiral perturbation theory (ChPT), they can also be calculated using one-loop calculations as was suggested already in [1], and proven in [5]. In contrast to renormalizable theories, in EFTs the LLog terms cannot be obtained simultaneously for all orders, and every order of the perturbative expansion requires an additional calculation. However, the evaluation of LLog terms is considerably simpler than a full calculation. As an example, the full two-loop leading logarithms in bosonic ChPT were known long before the full results [6].

Within bosonic EFTs the LLogs have been studied extensively. It has been shown that for EFTs with massless particles the LLog behavior is described by a closed set of equations with known kernels, which were elaborated in [7–10]. Although, the analytical solution of these equations is not known, one can generate numerically the first few hundreds of coefficients rather fast, and use the approximate numerical solution in applications. An example is the exploration of the "chiral inflation" of the pion radius within ChPT [11]. Taking into account the mass of the fields allows for non-zero tadpole diagrams, which leads to a rapidly increasing number of equations with the chiral order since one has to consider one-loop diagrams with an ever increasing number of external legs. Therefore, one needs to incorporate new processes at every new order. As a result, the difficulty of the calculation grows extremely fast with the chiral order. By automatizing the procedure for a large number of processes the LLogs are known up to seven loops for some quantities [12–15].

The main goal of this paper is to generalize the methods used for bosonic EFTs with masses to the nucleon case. As mentioned earlier, it is not only interesting from the theoretical side, but also necessary for the evaluation of nucleon parton distributions at $x \sim m_\pi/M_N$ [3,16]. In the paper we present the extension of the RG method of [5] to nucleon–pion ChPT. With its help, we calculate the LLog coefficients for the chiral expansion of the nucleon mass in the heavy-baryon formulation of ChPT. The main results are presented in Sections 6.3 and 6.4. An earlier application of LLogs in the nucleon sector was the calculation of the two-loop LLog contribution to the axial nucleon coupling constant g_A [17].

The paper is organized as follows: In Section 2 we introduce the concept of renormalization group order (RGO). This is needed since in the nucleon sector chiral counting and loop counting are not identical. Section 3 shows how the RGO concept works in the meson sector and quotes some known results. Section 4 introduces the heavy baryon ChPT Lagrangian in its two most common variants and the different meson parametrizations we have used as a check on our result. Section 5 shows how the RGO can be used to prove the calculation of the leading logarithms using only one-loop diagrams also in the nucleon sector. This is then used to calculate the LLogs for the nucleon mass in Section 6. Some technicalities are discussed in Sections 6.1 and 6.2. We then calculate the LLogs for the nucleon mass as well as the odd-power next-to-leading logarithms (NLLogs) in Section 6.3 up to four respectively five loops. The observed regularity in the leading logarithm allows to also calculate the five loop result with a mild assumption.