

# Casimir pistons with general boundary conditions

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## Abstract

In this work we analyze the Casimir energy and force for a scalar field endowed with general self-adjoint boundary conditions propagating in a higher dimensional piston configuration. The piston is constructed as a direct product  $I \times N$ , with  $I = [0, L] \subset \mathbb{R}$  and  $N$  a smooth, compact Riemannian manifold with or without boundary. The study of the Casimir energy and force for this configuration is performed by employing the spectral zeta function regularization technique. The obtained analytic results depend explicitly on the spectral zeta function associated with the manifold  $N$  and the parameters describing the general boundary conditions imposed. These results are then specialized to the case in which the manifold  $N$  is a  $d$ -dimensional sphere.

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## 1. Introduction

The Casimir effect refers to a broad set of phenomena which are caused by changes to the vacuum energy of a quantum field due to the presence of either boundaries or non-dynamical external fields. The effect was first predicted by Casimir in [12] who analyzed a configuration consisting of two parallel plates. However, in general the Casimir effect manifests itself through the appearance of a net force between two neutral objects [9]. Due to its theoretical as well as experimental importance the Casimir effect has been a subject of quite intense research for the past several decades (see, e.g., [8,9,12,22–24,36,40] and references therein). For the vast majority of configurations, calculations of the Casimir energy lead, unfortunately, to meaningless

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divergent quantities that require regularization and subsequent renormalization [4,10,20,21]. This can be accomplished through a number of techniques [9] which constitute rather standard tools in quantum field theory.

Piston configurations were first introduced in the well-known work of Cavalcanti [13] who analyzed the Casimir effect for a massless scalar field propagating in a rectangular box divided into two regions by a movable piston. He found that the Casimir energy generates a force that tends to move the piston to the closest wall. Since his seminal work, piston configurations have attracted widespread interest and have been the topic of a substantial number of publications. One of the main reasons that make piston configurations such an interesting subject of study lies in the fact that although their Casimir energy might be divergent, the corresponding Casimir force acting on the piston is well defined and free of divergences. This, however, becomes no longer generally true when one considers piston configurations with non-vanishing curvature [26–28].

Several types of geometric configurations for Casimir pistons have been considered throughout the literature, see e.g. [2,15–19,30,31,33–35]. Almost all of them, though, analyze the Casimir effect for quantum fields subject to standard boundary conditions. Here, by standard boundary conditions we intend Dirichlet, Neumann, and Robin boundary conditions. Other sets of boundary conditions that have also been considered are hybrid, or mixed, boundary conditions which are obtained by imposing different standard boundary conditions on different sides of the two chambers of the piston. The main purpose for studying such a wide variety of piston configurations endowed with standard boundary conditions is to analyze the dependence of the Casimir energy and force on the particular geometry of the system and on the boundary conditions imposed. Lately, repulsive Casimir forces have become a subject of major interest due to their relevance in the development of microelectromechanical devices (MEMS). As a result of their microscopic size such devices are afflicted by the problem of stiction, in which different components of the device adhere to each other due to an attractive Casimir force. It is, therefore, of particular importance to understand what type of boundary conditions, which are used to model properties of materials, need to be imposed in order to obtain a repulsive, or even vanishing, Casimir force. Configurations that lead to a repulsive Casimir force have been analyzed, for instance, in [11,25,37–39,42].

In this work we consider a massless scalar field propagating in a higher dimensional piston configuration endowed with general self-adjoint boundary conditions. Clearly, the general boundary conditions considered here contain, as particular cases, the standard and hybrid boundary conditions mentioned earlier. By using the spectral zeta function regularization technique we compute the Casimir energy of the piston configuration and the ensuing Casimir force on the piston itself. The expression we obtain for the Casimir force depends explicitly on the general boundary conditions imposed which, in turn, are described by six independent parameters. Our results are therefore suitable for analyzing how the Casimir force changes, both in magnitude and sign, when any of the parameters describing the general boundary conditions vary. The results obtained in this work can be used to determine a range of values for the parameters in the boundary conditions that results on a repulsive force of the piston from one or both ends of the piston configuration. These values provide a set of particular boundary conditions which could be utilized for selecting specific materials in the design and development of microelectromechanical devices. It is important to mention that the study of the Casimir energy for massless scalar fields endowed with general boundary conditions have been conducted, for instance, in [1]. Their analysis focuses on homogeneous parallel plates embedded in  $\mathbb{R}^D$ . For this configuration the variation of the Casimir force on the plates with respect to the general boundary conditions

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