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# Chiral symmetry breaking and chiral polarization: Tests for finite temperature and many flavors

Andrei Alexandru [a,*], Ivan Horváth [b]

[a] *The George Washington University, Washington, DC, USA*
[b] *University of Kentucky, Lexington, KY, USA*

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**Abstract**

It was recently conjectured that, in SU(3) gauge theories with fundamental quarks, *valence* spontaneous chiral symmetry breaking is equivalent to condensation of local dynamical chirality and appearance of chiral polarization scale $\Lambda_{ch}$. Here we consider more general association involving the low-energy layer of chirally polarized modes which, in addition to its width ($\Lambda_{ch}$), is also characterized by volume density of participating modes ($\Omega$) and the volume density of total chirality ($\Omega_{ch}$). Few possible forms of the correspondence are discussed, paying particular attention to singular cases where $\Omega$ emerges as the most versatile characteristic. The notion of *finite-volume "order parameter"*, capturing the nature of these connections, is proposed. We study the effects of temperature (in $N_f = 0$ QCD) and light quarks (in $N_f = 12$), both in the regime of possible symmetry restoration, and find agreement with these ideas. In $N_f = 0$ QCD, results from several volumes indicate that, at the lattice cutoff studied, the deconfinement temperature $T_c$ is strictly smaller than the overlap–valence chiral transition temperature $T_{ch}$ in real Polyakov line vacuum. Somewhat similar intermediate phase (in quark mass) is also seen in $N_f = 12$. It is suggested that deconfinement in $N_f = 0$ is related to indefinite convexity of absolute X-distributions.


* Corresponding author.
*E-mail address:* aalexan@gwu.edu (A. Alexandru).

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## 1. Introduction and summary

Eigensystems of Dirac operator in equilibrium gauge backgrounds carry the information on fermionic aspects of quark–gluon dynamics. As an important example, inspection of Dirac spectral representation for scalar fermionic density immediately reveals that spontaneous chiral symmetry breaking (SChSB) is equivalent to mode condensation of massless Dirac operator. Ever since this association has been pointed out [1], it became popular to think about SChSB in terms of quark near-zeromodes.

Contrary to SChSB, the mode condensation property is a well-defined notion in generic quark–gluon system, i.e. even when all quarks are massive. While there is no chiral symmetry of physical degrees of freedom to break in this case, condensing dynamics can still be described via chiral symmetry considerations. Indeed, one can introduce e.g. a pair of fictitious fermionic fields ("valence quarks") of degenerate mass $m_v$, and cancel their contribution to the action by also adding the associated bosonic partners [2]. This keeps the dynamics of physical quarks and gluons unchanged, but makes it meaningful to consider chiral rotations of valence fields in such extended system, and to inquire about "valence SChSB" in the $m_v \to 0$ limit. In this language,

$$\textbf{vSChSB} \iff \textbf{QMC} \tag{1}$$

i.e. quark mode condensation (QMC) in arbitrary quark–gluon system is equivalent to valence spontaneous chiral symmetry breaking (vSChSB): dynamics supports condensing modes if and only if it supports valence chiral condensate and valence Goldstone pions.

It is useful to think about SChSB in the above more general sense, especially when inquiring about the mechanism underlying the phenomenon [3]. Indeed, the response of massless valence quarks to gauge backgrounds of various quark–gluon systems provides a relevant point of dynamical distinction for associated theories: they either support "broken" or "symmetric" dynamics of the external massless probe. Moreover, valence SChSB is readily observed in lattice simulations with physically relevant flavor arrangements, and the associated dynamical characteristics change smoothly in the light quark regime [3]. Valuable lessons on SChSB can thus be learned by studying its valence version with massive dynamical quarks: vSChSB becomes SChSB as dynamical chiral limit is approached.

Unfortunately, the equivalence of quark mode condensation and valence SChSB does not provide window into specifics of broken quark dynamics. Indeed, the mode condensation property is merely a restatement of symmetry breakdown condition in Dirac spectral representation. However, it was recently proposed that another relation may hold, possibly with similar scope of validity, but with non-trivial dynamical connection to inner workings of the breaking phenomenon [3]. In particular, it was suggested that

$$\textbf{vSChSB} \iff \textbf{DChC} \tag{2}$$

i.e. that valence SChSB is equivalent to *dynamical chirality condensation* (DChC). This offers an intuitively appealing notion that the vacuum effect of chiral symmetry breaking is in fact the phenomenon of chirality condensation. In light of Eq. (1), the above relation carries the same information as QMC–DChC equivalence, which may be preferable for explicit checks.

While entities involved in the above relations will all be defined in Section 2, it should be pointed out now that DChC relates to *dynamical* notion of local chirality in modes [4]: it expresses the tendency for asymmetry in magnitudes of left–right components (local chiral polarization), measured with respect to the baseline of statistical independence. The associated quantifier, the correlation coefficient of polarization $C_A \in [-1, 1]$, is invariant with respect