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Integrals of motion in the many-body localized phase

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Abstract

We construct a complete set of quasi-local integrals of motion for the many-body localized phase of interacting fermions in a disordered potential. The integrals of motion can be chosen to have binary spectrum $\{0, 1\}$, thus constituting exact quasiparticle occupation number operators for the Fermi insulator. We map the problem onto a non-Hermitian hopping problem on a lattice in operator space. We show how the integrals of motion can be built, under certain approximations, as a convergent series in the interaction strength. An estimate of its radius of convergence is given, which also provides an estimate for the many-body localization–delocalization transition. Finally, we discuss how the properties of the operator expansion for the integrals of motion imply the presence or absence of a finite temperature transition.

1. Introduction

The thermodynamic description of macroscopic bodies, as shown by Boltzmann in his work on the foundations of statistical mechanics, is based on the assumption that the underlying

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microscopic dynamics are ergodic. More precisely, one assumes that the environment of any small subsystem of the macroscopic body acts as a thermal bath, with which the subsystem can exchange particles and energy, and which leads to the eventual thermalization of the subsystems, independently of its initial state. In order for thermalization to occur gradients in particle and energy density must be able to even out, which requires non-vanishing transport over arbitrarily large scales.

However, in the absence of interaction, Anderson [1] has shown that a sufficiently strong quenched disorder can localize quantum particles. This prevents the transport of energy and particles and therefore entails the non-ergodicity of the system. Already in Anderson's first paper, and later in the context of electron–electron interactions [2], it was surmised that this localization might persist in the presence of interactions, despite the widespread belief that any finite interactions would restore transport, ergodicity, and thus standard thermodynamic behavior, in such systems. Later, numerical investigations of the Hubbard model [3] hinted indeed at the possibility of such "many-body localization" (MBL), and more recently the seminal study of disordered electrons with weak short range interactions to all orders of perturbation theory provided important analytical insight into this phenomenon [4], predicting that in an isolated system, decoupled from any external bath, a finite interaction is required to induce delocalization and enable transport. Below this delocalization threshold, truly inelastic decay processes are impossible, as the system ceases to be a heat-bath for itself, and any d.c. transport is strictly absent. In this way many-body localized systems are crucially different from other situations where full ergodicity in phase space breaks down, such as in systems with spontaneously broken symmetries, one-dimensional integrable systems, or spin glasses. In all these examples, the thermal conductivity remains finite.[2] In contrast, a necessary[3] condition for many-body localization is the vanishing of the d.c. transport coefficients at non-zero temperature.

Since the seminal work by Basko, Aleiner and Altshuler [9,4,10], the paradigm of many-body localization has attracted a lot of interest, and the phenomenology of MBL phases and the localization transition have been explored, see for example [11–17]. Many-body localization opens the interesting possibilities of protection of topological order at finite temperature or of phase transitions below the equilibrium lower critical dimension [18–23]. It was even proposed that MBL could survive in the absence of quenched disorder [24–28]. (A different type of non-ergodic behavior, exhibiting, however, ballistic transport, has been conjectured in disorder free 1d systems that are close enough to integrability [29,30].)

Unambiguous experimental evidence of an MBL transition or phase is however still lacking at the time of writing, despite of promising developments [31,32].

An MBL phase can be seen as the prototype of a quantum glass phase, where the dynamics are slowed down indefinitely and where memory of the initial condition is retained in local observables for arbitrarily long times. This latter phenomenon has certain similarities with integrable systems [33,34], in which an extensive number of conserved quantities (integrals of motion) constrain the system to evolve in a much smaller submanifold than the one determined

[2] In integrable systems, transport of some quantities is often even more efficient than in non-integrable systems, being ballistic as opposed to diffusive.

[3] The condition is not sufficient, since even in the absence of diffusion thermalization might occur via sub-diffusive processes. This was found empirically in one-dimensional systems close enough to the localization transition [5,6]. Moreover, localization can occur also in time-dependent systems (e.g., periodically driven systems) that may have no conserved local densities and thus no meaningful d.c. transport [7,8]. For these systems, MBL is defined more generally as a phase where any local observable does not thermalize almost certainly.