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The double mass hierarchy pattern: Simultaneously understanding quark and lepton mixing

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Abstract

The charged fermion masses of the three generations exhibit the two strong hierarchies $m_3 \gg m_2 \gg m_1$. We assume that also neutrino masses satisfy $m_{\nu 3} > m_{\nu 2} > m_{\nu 1}$ and derive the consequences of the hierarchical spectra on the fermionic mixing patterns. The quark and lepton mixing matrices are built in a general framework with their matrix elements expressed in terms of the four fermion mass ratios, m_u/m_c, m_c/m_t, m_d/m_s and m_s/m_b, and m_e/m_μ, m_μ/m_τ, $m_{\nu 1}/m_{\nu 2}$ and $m_{\nu 2}/m_{\nu 3}$, for the quark and lepton sector, respectively. In this framework, we show that the resulting mixing matrices are consistent with data for both quarks and leptons, despite the large leptonic mixing angles. The minimal assumption we take is the one of hierarchical masses and minimal flavor symmetry breaking that strongly follows from phenomenology. No special structure of the mass matrices has to be assumed that cannot be motivated by this minimal assumption. This analysis allows us to predict the neutrino mass spectrum and set the mass of the lightest neutrino well below 0.01 eV. The method also gives the 1σ allowed ranges for the leptonic mixing matrix elements. Contrary to the common expectation, leptonic mixing angles are found to be determined solely by the four leptonic mass ratios without any relation to symmetry considerations as commonly used in flavor model building. Still, our formulae can be used to build up a flavor model that predicts the observed hierarchies in the masses — the mixing follows then from the procedure which is developed in this work.

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1. Introduction

The Standard Model of particle physics (SM) describes the interactions among elementary particles at high energies with great success. In spite of this, the setup of the SM lacks an explanation of the origin of fermion masses and mixing. In particular, for the quark sector, one observes six masses, three mixing angles and one phase. It is a simple exercise to relate the quark mixing matrix to the fundamental parameters of the theory, the Yukawa couplings. Generally, however, it is said that mixing angles as well as the masses are *independent* free parameters. Is there really no functional relation between the quark masses and the corresponding mixing matrix elements? There are many models in the literature that try to give an explanation of the mixing matrix elements in terms of the masses [1–28]. Most of them put assumptions on a specific texture in the original mass matrices. We shall show, by contrast, that the pure phenomenological observation of strong hierarchies in the quark masses leads to a functional description of the mixing matrix elements in terms of mass ratios. The consequences in the mixing of this phenomenological observation have already been studied [15,20,26,29–33]. Our approach differs from the previous ones in many aspects: (i) we take the Singular Value Decomposition of the complex mass matrices as a starting point offering a generic treatment for both quarks and leptons; (ii) by means of an approximation theorem we mathematically formulate the steps to build the reparametrization of the mixing matrix in terms of the singular values (fermion masses); (iii) we rotate the mass matrices in all three planes of family space whereas before, the 1–3 rotation was neglected; (iv) as the two unitary rotations in the 2–3 and 1–3 planes involve an approximation ($m_{f,1} = 0$ and $m_{f,2} = 0$, respectively) we consider for the first time a modified method of perturbation theory to add the effect of the terms neglected; (v) we do not consider the complex CP phases as free parameters and show that a minimal choice is sufficient to explain CP data; (vi) we provide explicit formulae for the mixing angles in terms of only mass ratios.

The applicability of this formulation to the leptonic mixing is not clear *a priori*. First, neutrino masses do not show any strong hierarchy, at best a very mild one. Second, the leptonic mixing matrix exhibits large mixing, while the one in the quark sector is rather close to the unit matrix. This picture seems to suggest two quite different origins for the respective mixing matrices: quark masses strongly dominating the mixing patterns, whereas geometrical factors found from symmetries shaping the leptonic mixing, with only a weak intervention from the lepton masses [34,35].

Fermion masses, on the other hand, are also as puzzling as the mixing matrices: the top quark mass is by far the largest among the charged fermions, there are six orders of magnitude separating the top quark from the electron mass, six orders of magnitude separating the largest neutrino mass from the electron mass (assuming a neutrino mass scale of 0.1 eV). There are three orders of magnitude between the masses of the up-type quarks, whereas two orders of magnitudes in the down-quark sector. Top and bottom quark are separated by two orders of magnitude — the lightest charged lepton and the heaviest quark by again six orders of magnitude. Within each (charged) fermion species ($f = u, d, e$), the masses follow a hierarchy $m_{f,3} \gg m_{f,2} \gg m_{f,1}$,

$$\begin{aligned} &m_u : m_c : m_t \approx 10^{-6} : 10^{-3} : 1, \qquad m_d : m_s : m_b \approx 10^{-4} : 10^{-2} : 1, \\ &m_e : m_\mu : m_\tau \approx 10^{-4} : 10^{-2} : 1, \end{aligned} \tag{1}$$

while the two squared mass differences measured from neutrino oscillations obey a much weaker hierarchy,

$$\Delta m^2_{21} : \Delta m^2_{31(32)} \approx 10^{-2} : 1. \tag{2}$$