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Strings, vortex rings, and modes of instability

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Abstract

We treat string propagation and interaction in the presence of a background Neveu–Schwarz three-form field strength, suitable for describing vortex rings in a superfluid or low-viscosity normal fluid. A circular vortex ring exhibits instabilities which have been recognized for many years, but whose precise boundaries we determine for the first time analytically in the small core limit. Two circular vortices colliding head-on exhibit stronger instabilities which cause splitting into many small vortices at late times. We provide an approximate analytic treatment of these instabilities and show that the most unstable wavelength is parametrically larger than a dynamically generated length scale which in many hydrodynamic systems is close to the cutoff. We also summarize how the string construction we discuss can be derived from the Gross–Pitaevskii Lagrangian, and also how it compares to the action for giant gravitons.

1. Introduction

We are interested in the dynamics of vortex rings in a medium, moving slowly relative to the speed of sound c_s and interacting with themselves through perturbations of the medium. We will use the following action to describe the interacting vortices:

$$S = \sum_\alpha \left[-c_s \tau_{1,\text{bare}} \int_{\Sigma_\alpha} dt\, d\theta |\partial_\theta \vec{X}_\alpha| + \mu_1 \int_{\Sigma_\alpha} B_2 \right] - \frac{\lambda}{2} \sum_{\alpha,\beta} \int_{\text{reg}} dt\, d\theta\, d\tilde{\theta} \frac{\partial_\theta \vec{X}_\alpha \cdot \partial_{\tilde{\theta}} \vec{X}_\beta}{|\vec{X}_\alpha(\theta) - \vec{X}_\beta(\tilde{\theta})|}, \tag{1}$$

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where α and β are labels running over the several separate vortices and B_2 is the pull-back of a spacetime gauge potential B_2 satisfying

$$dB_2 = H_3 = \frac{\rho_0}{6}\epsilon_{mnp}dx^m \wedge dx^n \wedge dx^p. \tag{2}$$

Essentially this action (but without the explicit tension term) was justified in [1] as an effective description of hydrodynamical vortices. Related actions were considered in the early literature on string theory, for example [2–4]. The tension term in (2) is understood to represent microscopic dynamics of the vortex core over which we do not have full control. The regularized integral $\int_{\text{reg}}$ provides some ultraviolet cutoff for the divergence that arises when the denominator of the integrand vanishes. A common choice of regulator, which we will adopt, is to replace

$$\left|\vec{X}_\alpha(\theta) - \vec{X}_\beta(\tilde{\theta})\right| \to \sqrt{\left(\vec{X}_\alpha(\theta) - \vec{X}_\beta(\tilde{\theta})\right)^2 + a^2}, \tag{3}$$

where the cutoff a is approximately the radius of the vortex core. We will assume $\mu_1 > 0$, which corresponds to a choice of orientation of the vortex; and it can be shown in the process of deriving (1) that $\lambda > 0$.

The action (1) can be derived as the quasi-static approximation of classical effective string dynamics, where the effective strings move in response to a strong spatial background H_3 and interact with themselves through the exchange of electrical components B_2. This classical effective string dynamics can in turn be derived from the Gross–Pitaevskii equation, under some simplifying assumptions and approximations. There is in addition a weak coupling to a radiation field which can be represented as a perturbation b_{ij} of B_2 and which propagates at the speed of sound.

Dynamics similar to (1) have been studied for over a hundred years. A notable early work is [5], and modern reviews include [6–9]. We will start in Section 2 by reviewing the instability of circular vortices [10]. We also calculate the zero point energy of fluctuations around circular vortices when it is well defined. We will continue in Section 3 by treating the stronger instabilities that arise in head-on collisions of circular vortices [11]. In both analyses we restrict ourselves to the limit of vanishingly small core size, so that we do not need to consider deformations of the core. Such deformations are believed to play an important role in quantitatively accurate descriptions of both single vortex instabilities [12–14] and the head-on collisions [8] in hydrodynamical settings. A novelty of our treatment is that in the small core limit we achieve full analytical control over both the unperturbed solutions and their linearized perturbations in terms of elliptic integrals.

The relation between vortices and classical strings has received significant attention in the string theory and cosmology literature. Early works [2–4] emphasized the possible relevance to superfluid Helium, proposed a cosmological role for vortex defects (cosmic strings) in theories with broken global $U(1)$ symmetry, uncovered the role of the Neveu–Schwarz field B_2, and arrived at essentially the dynamics (1), including the tension term and a renormalization of it due to the regulated interaction term. Derivations of the dynamics (1) from effective theories of superfluids can be found in [15,16]; see also [17] and the later work [18]. For the sake of completeness, we will review in Section 4 a derivation of (1) from the Gross–Pitaevskii action. We then conclude in Section 5 with a summary of results and a comparison of vortex ring phenomena to giant gravitons. Appendix A is devoted to a detailed comparison of single vortex results to an earlier study [10].