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Two-loop electroweak threshold corrections to the bottom and top Yukawa couplings

Bernd A. Kniehl *, Oleg L. Veretin

II. Institut für Theoretische Physik, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany

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Abstract

We study the relationship between the $\overline{\text{MS}}$ Yukawa coupling and the pole mass for the bottom and top quarks at the two-loop electroweak order $\mathcal{O}(\alpha^2)$ in the gaugeless limit of the standard model. We also consider the $\overline{\text{MS}}$ to pole mass relationships at this order, which include tadpole contributions to ensure the gauge independence of the $\overline{\text{MS}}$ masses. In order to suppress numerically large tadpole contributions, we propose a redefinition of the running heavy-quark mass in terms of the $\overline{\text{MS}}$ Yukawa coupling. We also present Δr in the $\overline{\text{MS}}$ scheme at $\mathcal{O}(\alpha^2)$ in the gaugeless limit. As an aside, we also list the exact two-loop expression for the mass counterterms of the bottom and top quarks.

1. Introduction

The recent discovery of the Higgs boson [1] was a giant leap in particle physics. It confirmed that the concept of spontaneous symmetry braking in connection with the generation of masses by the Higgs mechanism could be realized in nature. This does not explain, however, the very large spread of the fermion masses and the values of the masses themselves. It is generally believed that some grand unified theories could provide a solution to this fundamental problem.

* Corresponding author.

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An essential rôle in such analyses is played by the renormalization group (RG) equations, which determine the scale dependencies of the running parameters.

Due to the large values of their masses, the bottom quark and, even more so, the top quark attract great interest. Even disregarding the fact that quark masses require special consideration because quarks do not appear as free particles, one may introduce different parameters to describe the notion of quark mass. The most important definitions are those of the pole mass M, the running mass $m(\mu)$ in the modified minimal-subtraction ($\overline{\rm MS}$) scheme, and the running Yukawa coupling $y(\mu)$, defined in $\overline{\rm MS}$ scheme as well. Here, μ is the 't Hooft mass of dimensional regularization. The relationships between these quantities can be obtained in perturbation theory as series in the strong-coupling constant α_s and Sommerfeld's fine-structure constant α. The terms containing $\ln \mu^2$ can be obtained using the RG beta functions and anomalous dimensions. In QCD, they have been computed in the three- [2] and four-loop [3] approximations. In the standard model (SM), the corresponding RG functions, known through two loops since Ref. [4], have recently been evaluated at the three-loop level [5].

The other aspect of the problem is the matching between the running parameters and the physical observables. These so-called threshold relationships include not only terms with $\ln \mu^2$, but also terms of non-logarithmic type. The relation between the $\overline{\rm MS}$ mass and the pole mass of a quark was elaborated in QCD at one [6], two [7], and three [8] loops. The two-loop result in the supersymmetric extension of QCD was obtained in Ref. [9]. These corrections can be readily applied also to the Yukawa coupling of a quark. However, the situation becomes more complicated if electroweak corrections are taken into account. In this case, the relation between the $\overline{\rm MS}$ mass and the $\overline{\rm MS}$ Yukawa coupling of a fermion becomes nontrivial. The full one-loop corrections to the relationships between the $\overline{\rm MS}$ Yukawa couplings and the pole masses of the fermions were derived in Ref. [10]. The mixed QCD-electroweak two-loop corrections were evaluated for the top quark in Ref. [11] and for the other fermions in Ref. [12] by using the method of large-mass expansion. In some earlier calculations, the tadpole contributions were omitted in the $\overline{\rm MS}$ to pole mass relations of fermions, e.g. in the calculation of the electroweak parameter $\Delta\rho$ in the gaugeless limit in Ref. [13]. Expressed in terms of the top-quark and Higgs-boson pole masses, the result for $\Delta\rho$ thus obtained is correct, but some intermediate results differ from those in Ref. [11]. This situation was clarified in Ref. [14].

The aim of the present paper is to extend the theoretical knowledge of the relationships between the $\overline{\rm MS}$ masses and Yukawa couplings and the pole masses of the bottom and top quarks by calculating the two-loop electroweak corrections, at order $O(\alpha^2)$, in the approximation provided by the gaugeless limit.

This paper is organized as follows. In Sections 2 and 3, we give all the necessary definitions concerning the $\overline{\rm MS}$ mass and Yukawa coupling, respectively. In Sections 4 and 5, we list analytical results for the bottom and top $\overline{\rm MS}$ Yukawa couplings, respectively. In Section 6, we present our numerical analysis. In Appendix A, we list two-loop expressions for the electroweak parameter $\Delta\bar{r}$, which enters the relationships between the $\overline{\rm MS}$ masses and Yukawa couplings, both in the gaugeless and heavy-top-quark limits. Appendix B contains the exact two-loop renormalization constants of the bottom- and top-quark masses in the $\overline{\rm MS}$ scheme. In Appendix C, we present the $\overline{\rm MS}$ to pole mass relation for the bottom quark in the heavy-top-quark limit.