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Position space formulation for Dirac fermions on honeycomb lattice

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Abstract

We study how to construct Dirac fermion defined on the honeycomb lattice in position space. Starting from the nearest neighbor interaction in tight binding model, we show that the Hamiltonian is constructed by kinetic term and second derivative term of three flavor Dirac fermions in which one flavor has a mass of cutoff order and the other flavors are massless. In this formulation, the structure of the Dirac point is simplified so that its uniqueness can be easily shown even if we consider the next-to-nearest neighbor interaction. We also show that there is a hidden exact $U(1)$ symmetry (flavor–chiral symmetry) at finite lattice spacing, which protects the masslessness of the Dirac fermion, and discuss the analogy with the staggered fermion formulation.

1. Introduction

Graphene forms from a layer of carbon atoms with hexagonal tiling [1–4] and it is much discussed in condensed matter physics as well as high energy physics for its remarkable features (see [5,6] and references therein). One of the most important features of graphene is that the

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quasiparticles behave like massless Dirac fermion with effective speed of light near $c/300$ [2,7]. An explanation to the question why massless Dirac fermion emerges in non-relativistic many body system was primarily given by Semenoff [8]. In this model, the low energy excitations around two independent Dirac points on the Fermi-surface is described by two relativistic Weyl fermions having opposite chiralities, which are also regarded as massless Dirac fermion.

Although Semenoff's model is remarkable for its peculiar feature, it is not the first case with exact massless Dirac fermion from the lattice. In lattice gauge theory, there are several formulations to describe Dirac fermion on the lattice. In the staggered fermion formulation [9], the 2^{d-2} flavor Dirac fermions emerge at low energy from a single spinless fermion hopping around the d-dimensional hypercubic lattice, in close analogy to the Semenoff's model. The emergence of the Dirac fermion in staggered fermion has been studied in momentum space [10] and in position space [11]. In the former case, the fermion field is divided into 2^d components corresponding to the subdomains in the total momentum space. In the latter case, 2^d spin–flavor degrees of freedom of the Dirac fermion arise from the sites within the d-dimensional hypercubic unit cell.

In the case of honeycomb lattice in $2+1$ dimension, Dirac fermion field has been defined as two excitations on different regions of Brillouin Zone (BZ) in the continuum space–time [8]. Since this approach is very similar to the momentum space formulation for staggered fermion, it is natural to expect that position space formulation might also be possible for graphene model. Since the position space formulation easily extends local gauge interacting theory, it enables us to implement the dynamical calculation of physical observables in Monte-Carlo simulation more straightforwardly [12,13]. Furthermore, this formulation also has the manifest structure of flavor symmetry of Dirac fermion field, and so that the quantum number of low energy excitations is clearly identified.

In this paper, we show how to construct the Dirac fermion in position space on honeycomb lattice. It may be useful to advance a study of the dynamical nature of graphene with numerical approaches using Monte-Carlo simulation [14–17]. This approach plays an important role for more rigorous discussion for than modeling one [18,19] (also see [20]).

This paper is organized as follows. In Section 2, we briefly review a momentum space formulation of Dirac fermion derived from tight-binding approximation of graphene model. In Section 3, after introducing the formulation in position space, we discuss uniqueness of Dirac point and existence of physical mode, and then in Section 4, we show that two massless Dirac fermions appear at low energy region. In Section 5, we discuss the exact flavor–chiral symmetry in our formulation. The last section is devoted to the summary and discussion.

2. The conventional derivation from honeycomb lattice

We first review the conventional derivation of Dirac fermion formulation from tight binding model of honeycomb lattice [8]. Let us start from the tight binding Hamiltonian

$$\begin{aligned}\mathcal{H} = &-t\sum_{\vec{r}\in\mathrm{A}}\sum_{i=1,2,3}\sum_{\sigma}\left[a_\sigma^\dagger(\vec{r})b_\sigma(\vec{r}+\vec{s}_i)+b_\sigma^\dagger(\vec{r}+\vec{s}_i)a_\sigma(\vec{r})\right]\\ &-t'\left[\sum_{\vec{r}\in\mathrm{A}}\sum_{j=1}^{6}\sum_{\sigma}a_\sigma^\dagger(\vec{r})a_\sigma\left(\vec{r}+\vec{s}'_j\right)+\sum_{\vec{r}\in\mathrm{B}}\sum_{j=1}^{6}\sum_{\sigma}b_\sigma^\dagger\left(\vec{r}+\vec{s}'_j\right)b_\sigma(\vec{r})\right],\end{aligned} \tag{1}$$

where the first line is the nearest neighbor hopping term and the second line is the next-to-nearest neighbor hopping term, and t, t' are hopping amplitudes. $a(a^\dagger)$ and $b(b^\dagger)$ are the fermionic annihilation (creation) operators of electrons on two triangular sublattices A and B respectively