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Nuclear Physics B 885 (2014) 493–504

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

Incarnations of instantons

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Received 18 February 2014; accepted 28 May 2014

Available online 2 June 2014

Editor: Stephan Stieberger

Abstract

Yang–Mills instantons in a pure Yang–Mills theory in four Euclidean space can be promoted to particle-like topological solitons in $d = 4+1$ dimensional space–time. When coupled to Higgs fields, they transform themselves in the Higgs phase into Skyrmions, lumps and sine-Gordon kinks, when trapped inside a non-Abelian domain wall, non-Abelian vortex and monopole string, respectively. Here, we point out that a closed monopole string, non-Abelian vortex sheet and non-Abelian domain wall in S^1, S^2 and S^3 shapes, respectively, are all Yang–Mills instantons if their S^1, S^2 and S^3 moduli, respectively, are twisted along their world-volumes.

1. Introduction

Recent discovery of non-Abelian vortices [1,2] and non-Abelian domain walls [3–5] has been revealing relations among different topological solitons in diverse dimensions [6–8]. When a 't Hooft–Polyakov monopole [9] is put into the Higgs phase, the magnetic fluxes from it are squeezed into the form of magnetic vortices, becoming a confined monopole [10–13]. This configuration can be regarded as a kink inside a vortex. In other words, the monopole turns to the kink when it resides in the vortex. In the Higgs phase, Yang–Mills instantons are unstable to shrink in the bulk. However, they can stably exist as lumps (or sigma model instantons) [14] when trapped inside a non-Abelian vortex [12,15], while they can stably exist as Skyrmions when trapped inside a non-Abelian domain wall [4]. Recently, it has been also found that instantons transform themselves into sine-Gordon kinks when trapped inside a monopole string in a certain situation [16]. Vortices or lumps become sine-Gordon kinks [17–19] when trapped

http://dx.doi.org/10.1016/j.nuclphysb.2014.05.032

Table 1
Host solitons of trapped instantons in $d = 2+1$ (a) and in $d = 4+1$ (b)–(d). The shape of the world-volume can be non-compact $\mathbb{R}^n$ or compact S^n, corresponding to trapped and untrapped instantons, respectively. NA denotes "non-Abelian".

Host solitons	Bulk dim	Codim.	Moduli	w.v. shape	w.v. soliton	Homotopy
(a) $\mathbb{C}P^1$ domain wall	$2+1$	1	S^1	$\mathbb{R}^1$ or S^1	SG kink	$\pi_1(S^1)$
(b) NA domain wall		1	S^3	$\mathbb{R}^3$ or S^3	Skyrmion	$\pi_3(S^3)$
(c) NA vortex sheet	$4+1$	2	S^2	$\mathbb{R}^2$ or S^2	lump	$\pi_2(S^2)$
(d) monopole string		3	S^1	$\mathbb{R}^1$ or S^1	SG kink	$\pi_1(S^1)$

inside a $\mathbb{C}P^1$ domain wall [20,21]. Skyrmions become lumps or baby Skyrmions [22,23] inside a non-Abelian domain wall [24], and more generally N dimensional Skyrmions become $N-1$ dimensional Skyrmions inside a non-Abelian domain wall [25]. Among those, one of the most successful applications to field theory may be made by confined monopoles [11,12] which explain the coincidence of BPS spectra in $3+1$ and $1+1$ dimensions [26].

The $\mathbb{C}P^1$ model in $d = 1+1$ dimensions offers a toy model of Yang–Mills theory in $d = 3+1$ dimensions. The presence of instantons [14] is one of such similarities between them. Sigma model instantons can be promoted to lumps in $d = 2+1$ dimensions. With a mass term admitting two vacua, the $\mathbb{C}P^1$ model allows a domain line with a $U(1)$ modulus [20,21]. If the $U(1)$ modulus winds around a straight domain line, a sine-Gordon kink is formed on it corresponding to a lump in the bulk [17], which is a lower dimensional analogue of trapped instantons. The dynamics of such domain wall Skyrmions were studied [19].

Then, a question arises. Do all instantons have to be trapped into some host solitons to become composite states in theories in the Higgs phase? The answer is no. If one makes a closed domain line with the $U(1)$ modulus wound along it, such a twisted closed domain line is nothing but an isolated lump [27]. This is stabilized against shrinkage and becomes a baby Skyrmion if one adds a four derivative Skyrme term in the original theory [18]. An alternative way to stabilize a twisted closed domain line is to give a linear time dependence on the $U(1)$ modulus, which results in a Q-lump [28]. This situation is summarized in Table 1(a) and is illustrated in Fig. 1(a). Thus, we have both trapped and untrapped instantons, where the domain line world-volume is $\mathbb{R}^1$ and S^1, respectively. We may call the latter as incarnations of instantons.

Here, we propose higher dimensional analogues of this phenomenon for Yang–Mills instantons in Yang–Mills–Higgs theories in $d = 4+1$ dimensions. In this dimensionality, instantons are particle-like solitons, while a vortex and monopole are a sheet (membrane) and string, respectively. In order to demonstrate our idea, we take the gauge group as $U(2) = [SU(2) \times U(1)]/\mathbb{Z}_2$ but generalizations to $U(N) = [SU(N) \times U(1)]/\mathbb{Z}_N$ or other groups are straightforward, since vortices in arbitrary gauge groups [29] such as $SO(N)$ and $USp(2N)$ [30] were already constructed. We put the system into the Higgs phase where the $U(2)$ gauge group is spontaneously broken completely, by introducing some doublet Higgs fields with the common $U(1)$ charges. Unlike the case of $d = 2+1$, there are three possibilities of incarnations of instantons. In Table 1(b)–(d), we summarize host solitons with world-volume $\mathbb{R}^{n,1}$ in which instantons can reside stably, *i.e.*, a non-Abelian domain wall ($n = 3$), non-Abelian vortex sheet ($n = 2$), and monopole string ($n = 1$) of codimensions one, two and three, respectively. These solitons have internal moduli S^3, S^2 and S^1, respectively, as localized Nambu–Goldstone zero modes in addition to translational moduli. When these moduli wind in the spatial world-volumes $\mathbb{R}^n$ of the host solitons according to the homotopy groups $\pi_n(S^n) \simeq \mathbb{Z}$, there appear Skyrmions, lumps and sine-Gordon kinks in the world-volumes of the domain wall, vortex sheet, and monopole string,