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ScienceDirect
Nuclear Physics B 885 (2014) 679–712
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

Chiral primaries in strange metals

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Received 29 April 2014; accepted 9 June 2014

Available online 11 June 2014

Editor: Verlinde Herman

Abstract

It was suggested recently that the study of 1-dimensional QCD with fermions in the adjoint representation could lead to an interesting toy model for strange metals and their holographic formulation. In the high density regime, the infrared physics of this theory is described by a constrained free fermion theory with an emergent $\mathcal{N} = (2, 2)$ superconformal symmetry. In order to narrow the choice of potential holographic duals, we initiate a systematic search for chiral primaries in this model. We argue that the bosonic part of the superconformal algebra can be extended to a coset chiral algebra of the form $\mathcal{W}_N = \mathrm{SO}(2N^2 - 2)_1/\mathrm{SU}(N)_{2N}$. In terms of this algebra the spectrum of the low energy theory decomposes into a finite number of sectors which are parametrized by special necklaces. We compute the corresponding characters and partition functions and determine the set of chiral primaries for $N \leq 5$.

1. Introduction

Low dimensional examples of dualities between conformal field theories and gravitational models in Anti-deSitter (AdS) space provide an area of active research. There are several reasons why such developments are interesting. On the one hand, many low dimensional critical theories can actually be realized in condensed matter systems. As they are often strongly coupled, the AdS/CFT correspondence might provide intriguing new analytic tools to compute relevant physical observables. On the other hand, low dimensional incarnations of the AdS/CFT

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http://dx.doi.org/10.1016/j.nuclphysb.2014.06.004
0550-3213/

correspondence might also offer new views on the very working of dualities between conformal field theories and gravitational models in AdS backgrounds. This applies in particular to the AdS_3/CFT_2 correspondence since there exist many techniques to solve 2-dimensional models directly, without the use of a dual gravitational theory. Recent examples in this direction include the correspondence between certain 2-dimensional coset conformal field theories and higher spin gauge theories [1,2], see also [3–6] for examples involving supersymmetric conformal field theories and [8] for a more extensive list of the vast literature on the subject. It would clearly be of significant interest to construct new examples of the AdS_3/CFT_2 correspondence which involve full string theories in AdS_3.

In 2012, Gopakumar, Hashimoto, Klebanov, Sachdev and Schoutens [7] studied a two-dimensional adjoint QCD in which massive Dirac fermions Ψ are coupled to an $SU(N)$ gauge field. The fermions were assumed to transform in the adjoint rather than the fundamental representation of the gauge group. In the strongly coupled high density region of the phase space, the corresponding infrared fixed point is known to develop an $\mathcal{N} = (2,2)$ superconformal symmetry. For gauge groups SU(2) and SU(3) the fixed points possess Virasoro central charge $c_2 = 1$ and $c_3 = 8/3$, respectively. These central charges are smaller that the critical value of $c = 3$ below which one can only have a discrete set of $\mathcal{N} = (2,2)$ superconformal minimal models. Such theories are very well studied. Once we go beyond $N = 3$, however, the central charge $c_N = (N^2 - 1)/3$ of the infrared fixed point exceeds the critical value and the models are very poorly understood at present. Note that the central charge c_N of these models grows quadratically with the rank $N - 1$ of the gauge group. While this is very suggestive of a string theory dual, there exist very little further clues on the appropriate choice of the 7-dimensional compactification manifold M^7 of the relevant AdS background.

The most interesting structure inside any $\mathcal{N} = (2,2)$ superconformal field theory is its chiral ring. Recall that the $\mathcal{N} = (2,2)$ superconformal algebra contains a U(1) R-charge Q. The latter provides a lower bound on the conformal weights h in the theory, i.e. physical states ϕ in a unitary superconformal field theory obey the condition $h(\phi) \geq Q(\phi)$. States in the Neveu–Schwarz sector that saturate this bound, i.e. for which $h(\phi) = Q(\phi)$, are called *chiral primaries*. Since chiral primaries are protected by supersymmetry, they are expected to play a key role in discriminating between potential gravitational duals for the infrared fixed point of adjoint QCD. More concretely, the space of chiral primaries in the limit of large N should carry essential information on the compactification manifold M^7 of the dual AdS_3 background.

The goal of our work is to initiate a systematic study of the chiral ring for the models proposed by Gopakumar et al. In [7] the partition function of the infrared fixed point was studied for $N = 2, 3$. In these two cases the chiral ring is well understood through the relation with $\mathcal{N} = (2,2)$ minimal models, as we mentioned above. The chiral primaries that are found in these two simple models are special representatives of a larger class of *regular* chiral primaries that can be constructed for all N. But once we pass the critical value of the central charge, i.e. for $c_N > 3$, additional chiral primaries start showing up. We shall find one example at $N = 4$ and three non-regular, or *exceptional*, chiral primaries for $N = 5$. In order to do so, we develop some technology that can be applied also to larger values of N and we hope that it will provide essential new tools in order to address the large N limit.

Let us briefly discuss the plan of this paper. In the next section, we shall describe the low energy theory, identify its chiral algebra, construct the relevant modular invariant partition function and finally discuss the emergent $\mathcal{N} = (2,2)$ superconformal symmetry. Our discussion differs a bit from the one in [7] in that we work with a larger chiral algebra. Our algebra has the advantage that it contains the R-current of the model. This gives us more control over chiral primaries in