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Boundary effects on the supersymmetric sine-Gordon model through light-cone lattice approach

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Abstract

We discussed subspaces of the $\mathcal{N} = 1$ supersymmetric sine-Gordon model with Dirichlet boundaries through light-cone lattice regularization. In this paper, we showed, unlike the periodic boundary case, both of Neveu–Schwarz (NS) and Ramond (R) sectors of a superconformal field theory were obtained. Using a method of nonlinear integral equations for auxiliary functions defined by eigenvalues of transfer matrices, we found that an excitation state with an odd number of particles is allowed for a certain value of a boundary parameter even on a system consisting of an even number of sites. In a small-volume limit where conformal invariance shows up in the theory, we derived conformal dimensions of states constructed through the lattice-regularized theory. The result shows existence of the R sector, which cannot be obtained from the periodic system, while a winding number is restricted to an integer or a half-integer depending on boundary parameters.

1. Introduction

Physical systems on finite volume show interesting features such as edge states and boundary critical exponents and their importance has been noticed for years. It is also important, as any real materials are finite-size systems, to know boundary effects on physical quantities. Nevertheless,

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existence of boundaries often destroys good symmetry obtained for periodic systems, which makes it more difficult to study a system with boundaries.

For this reason, it would be nice to work on systems with good symmetries, even after adding non-trivial boundary conditions, which somehow allow us exact calculation of physical quantities. Although adding boundaries breaks symmetry of an integrable system at boundaries, whose integrability is ensured by the Yang–Baxter equation, there exist such boundary conditions that preserve integrability of the system satisfying the reflection relation [1,2] at boundaries. Due to the Yang–Baxter equation and the reflection relation, a many-body scattering process can be decomposed into a sequence of two-body scatterings which allows us to find exact scattering and reflection matrices.

An example which holds these symmetries is the spin-$\frac{1}{2}$ XXZ spin chain with boundary magnetic fields, whose R- and K-matrices can be obtained as solutions of the Yang–Baxter equation and the reflection relation. Another example is the sine-Gordon (SG) model with Dirichlet boundary conditions, which is obtained through bosonization of the spin-$\frac{1}{2}$ XXZ spin chain with boundary magnetic field. Both models has characterizing R-matrices associated with the $U_q(sl_2)$-algebra [3,4].

Different methods have been developed for spin chains and quantum field theories, since the former model is a discrete system, while the latter a continuum one. For spin chains, a transfer matrix method is often used to solve a system by regarding a two-dimensional lattice with time sequences of a transfer matrix. The Bethe-ansatz method is one of the most successful method to diagonalize a transfer matrix [5]. This method can be also applied to a system with non-trivial boundaries, as long as they satisfy the reflection relation. For instance, the XXZ model with boundary fields was first solved by the coordinate Bethe-ansatz method [6] and the method was algebraically formulated for the diagonal boundary case by introducing the double-row transfer matrix [2].

In a presence of magnetic boundary fields, existence of boundary bound states have been found through a q-deformed vertex operator [7] and later also by the Bethe-ansatz method [8, 9]. In a realm of the Bethe-ansatz method, boundary bound states are obtained as imaginary solutions of the Bethe-ansatz equations [8,9]. One needs exact distribution of Bethe roots for computation of physical quantities by the Bethe-ansatz method. Existence of imaginary roots slightly deforms root density for the bulk, and as a result deforms root distribution for the ground state as well. This fact leads us to a question whether boundary bound states are to be included in the ground state or not. The answer to this question was given for the repulsive regime [8] and for the attractive regime [9] by calculating a energy shift coming from emergence of imaginary roots themselves and a shift of root density driven by imaginary roots.

On the other hand, analytical discussion of a continuum theory has been achieved by the bootstrap approach [10]. This method allows us to compute a scattering matrix between any particles subsequently from a soliton–soliton S-matrix obtained as a solution of the Yang–Baxter equation. Similarly, the boundary bootstrap principle was also developed which subsequently gives a reflection amplitude on a boundary with excitation particles.

In the context of a quantum field theory, boundary bound states are obtained as poles in a reflection matrix. Existence of boundary bound states in the SG model with Dirichlet boundary conditions was discussed in [11,12] together with explicit forms of reflection matrices. Then spectrum of boundary states has been calculated in [13–16]. However, it is hard to know whether boundary bound states are included in the ground state or not, since in a quantum field theory realm, the ground state is always considered as a vacuum.