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Non-perturbative Nekrasov partition function from string theory

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Abstract

We calculate gauge instanton corrections to a class of higher derivative string effective couplings introduced in [1]. We work in Type I string theory compactified on K3 × T^2 and realise gauge instantons in terms of D5-branes wrapping the internal space. In the field theory limit we reproduce the deformed ADHM action on a general Ω-background from which one can compute the non-perturbative gauge theory partition function using localisation. This is a non-perturbative extension of [1] and provides further evidence for our proposal of a string theory realisation of the Ω-background.

1. Introduction

Gauge theory instantons have a natural description as bound states of D-branes in string theory [2,3]. This opens the possibility to calculate non-perturbative corrections in field theory from

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tree-level string amplitudes. Since the former are notoriously difficult to compute, whereas established techniques exist to obtain the latter, this provides very interesting insights into the inner structure of field theories and has lead to remarkable developments over the recent years. Furthermore, the series of works [4–7] that led to the partition function of supersymmetric gauge theories in the Ω-background has triggered considerable interest on this connection.

Indeed, four-dimensional super-Yang–Mills theory can be realised on a stack of D3-branes in Type IIB string theory, with additional D(-1)-branes playing the role of instantons. Their corrections to the Yang–Mills action are captured by string disc diagrams with boundary field insertions. In [8] the effect of a non-trivial constant string background in this setup was considered, by including additional bulk vertices in the tree-level amplitudes. It was shown that the insertion of anti-self-dual graviphoton field strength tensors in the point particle limit correctly reproduces the ADHM action on an Ω-background with one of its deformation parameters switched off (*e.g.* $\epsilon_+ = 0$). Using localisation techniques [4–7,9–11] this allows one to compute the non-perturbative Nekrasov partition function $Z^{\text{Nek}}(\epsilon_+ = 0, \epsilon_-)$.

Obtaining the partition function for a gauge theory on a generic Ω-background (with $\epsilon_+ \neq 0$) from string theory remains an interesting question. Phrased differently, one would like to find a modification of the anti-self-dual graviphoton background, considered in [8], giving rise to the fully deformed ADHM action in the point-particle limit of the appropriate disc diagrams. A hint for answering this question comes from a series of higher derivative one-loop couplings in the effective action of the Heterotic string compactified on K3 $\times\, T^2$, considered in [1]. These terms generalise a class of BPS-saturated couplings of gravitons and anti-self-dual graviphoton field strength tensors through additional couplings to the self-dual field strength tensor $F_{\bar{T}}$ of the vector partner of the Kähler modulus of the internal T^2. In the field theory limit, these one-loop amplitudes precisely reproduce the perturbative contribution of the Nekrasov partition function on the full Ω-background, *i.e.* with $\epsilon_-, \epsilon_+ \neq 0$. We therefore expect that, including $F^{\bar{T}}_{(+)}$ also as a background field in the instanton computation described above, allows us to extract the fully deformed ADHM action from string theory. In this paper, we show that this is indeed the case.

We work in Type I string theory compactified on K3 $\times\, T^2$ and consider D9-branes together with D5-instantons wrapping K3 $\times\, T^2$. This setting is dual to Heterotic string theory on K3 $\times\, T^2$ and the corresponding background is given by anti-self-dual graviphotons and self-dual field strength tensors of the vector partner of $\bar{S}'$, which we refer to as $\bar{S}'$-vectors in the remainder of this work. We compute all tree-level diagrams with boundary insertions that couple non-trivially to this background which, in the field theory limit, correctly reproduce the fully Ω-deformed version of the ADHM action, which was used to compute Nekrasov's partition function [6,7].

This result can also be interpreted as computing gauge theory instanton corrections to the higher derivative couplings discussed in [1]. The fact that we reproduce precisely the full Nekrasov partition function can also be seen as further evidence for the proposal that these couplings furnish a worldsheet description of the refined topological string. Indeed, our results suggest that the background introduced in [1] can be understood as a physical realisation of the Ω-background in string theory.

We would like to mention that various RR backgrounds were recently discussed in [12,13] where the Ω-deformed ADHM action was recovered using the language of D-instantons. However, contrary to our present work, the backgrounds considered there do not involve NS–NS field strengths. Moreover, the interpretation in terms of a string effective coupling, as well as the perturbative contribution, were not considered.

The paper is organised as follows. In Section 2, we briefly review the construction of gauge theory instantons in Type I as a D5–D9 system and set the notation for the various moduli arising