Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B 880 (2014) 134–154

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

Covariant action for a string in *doubled yet gauged* spacetime

Kanghoon Lee [a,b], Jeong-Hyuck Park [c,d,*]

[a] *Blackett Laboratory, Imperial College, London SW7 2AZ, England, UK*
[b] *Center for Quantum Spacetime, Sogang University, Seoul 121-742, Republic of Korea*
[c] *Department of Applied Mathematics and Theoretical Physics, Cambridge, CB3 0WA, England, UK*
[d] *Department of Physics, Sogang University, Seoul 121-742, Republic of Korea* [1]

Received 16 December 2013; received in revised form 6 January 2014; accepted 7 January 2014

Available online 9 January 2014

Abstract

The section condition in double field theory has been shown to imply that a physical point should be one-to-one identified with a gauge orbit in the doubled coordinate space. Here we show the converse is also true, and continue to explore the idea of *spacetime being doubled yet gauged*. Introducing an appropriate gauge connection, we construct a string action, with an arbitrary generalized metric, which is completely covariant with respect to the coordinate gauge symmetry, generalized diffeomorphisms, world-sheet diffeomorphisms, world-sheet Weyl symmetry and $\mathbf{O}(D,D)$ T-duality. A topological term previously proposed in the literature naturally arises and a self-duality condition follows from the equations of motion. Further, the action may couple to a T-dual background where the Riemannian metric becomes everywhere singular.

Keywords: String; T-duality; Double Field Theory

* Corresponding author at: Department of Physics, Sogang University, Seoul 121-742, Republic of Korea.
E-mail addresses: kanghoon.lee@imperial.ac.uk (K. Lee), park@sogang.ac.kr (J.-H. Park).

1 Sabbatical leave of absence.

http://dx.doi.org/10.1016/j.nuclphysb.2014.01.003

1. Introduction

In order to realize $\mathbf{O}(D,D)$ T-duality as a manifest symmetry [1–8], Double Field Theory (DFT) [9–14] doubles the spacetime dimension, from D to $D+D$, with doubled coordinates, $x^A = (\tilde{y}_\mu, y^\nu)$, of which the first and the last correspond to the 'winding' and the 'ordinary' coordinates respectively. However, DFT is not truly doubled since all the fields – including any local symmetry parameters – are subject to the section condition: the $\mathbf{O}(D,D)$ invariant d'Alembertian operator must be trivial, acting on arbitrary fields,

$$\partial_A \partial^A \Phi(x) = 0, \tag{1.1}$$

as well as their products, or equivalently

$$\partial_A \Phi_1(x) \partial^A \Phi_2(x) = 0. \tag{1.2}$$

While the section condition might appear somewhat odd or even invidious from the conventional Riemannian point of view, it is readily satisfied when all the fields are, up to $\mathbf{O}(D,D)$ rotations, independent of the dual winding coordinates, i.e. $\frac{\partial}{\partial \tilde{y}_\mu} \equiv 0$. This kind of explicit 'section-fixing' reduces DFT to *generalized geometry* [15–24] where the spacetime is not enlarged and the duality is less manifest.

Much progress has been made in recent years based on the notion of doubled spacetime subject to the section condition [25–59], including the state of the art reviews [52,54] and the construction of $\mathcal{N}=2$ $D=10$ maximally supersymmetric double field theory [46] as the unification of type IIA and IIB supergravities.[2] Analogous parallel developments on U-duality are also available [60–73] [3] which may all be incorporated into the *grand scheme of* E_{11} [74–77].

The $(D+D)$-dimensional doubled spacetime is far from being an ordinary Riemannian manifold, since it postulates the existence of a globally well-defined $\mathbf{O}(D,D)$ invariant constant metric,

$$\mathcal{J}_{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{1.3}$$

which serves to raise and lower the doubled spacetime indices.[4] Further, the diffeomorphism symmetry is generated not by the ordinary Lie derivative but by a generalized one,

$$\begin{aligned} \hat{\mathcal{L}}_V T_{A_1\cdots A_n} := {} & V^B \partial_B T_{A_1\cdots A_n} + \omega \partial_B V^B T_{A_1\cdots A_n} \\ & + \sum_{i=1}^{n} (\partial_{A_i} V_B - \partial_B V_{A_i}) T_{A_1\cdots A_{i-1}}{}^{B}{}_{A_{i+1}\cdots A_n}, \end{aligned} \tag{1.4}$$

where ω is the weight of the DFT-tensor, $T_{A_1\cdots A_n}(x)$, and $V^A(x)$ is an infinitesimal diffeomorphism parameter, as a DFT vector field which must also obey the section condition,

$$\partial_A \partial^A V^B(x) = 0, \qquad \partial_A V^B(x) \partial^A \Phi(x) = 0. \tag{1.5}$$

The generalized Lie derivative of the $\mathbf{O}(D,D)$ metric vanishes for consistency,

---

2 Cf. http://strings2013.sogang.ac.kr//main/?skin=video_GS_2.htm.

3 Cf. http://strings2013.sogang.ac.kr//main/?skin=video_27_5.htm.

4 For example, $\partial^A = \mathcal{J}^{-1AB}\partial_B$ as done in (1.1) and (1.2).