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Nuclear Physics B 874 [FS] (2013) 594–619

NUCLEAR PHYSICS B

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Symmetry-improved CJT effective action

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Received 17 May 2013; accepted 6 June 2013

Available online 11 June 2013

Abstract

The formalism introduced by Cornwall, Jackiw and Tomboulis (CJT) provides a systematic approach to consistently resumming non-perturbative effects in Quantum Thermal Field Theory. One major limitation of the CJT effective action is that its loopwise expansion introduces residual violations of possible global symmetries, thus giving rise to massive Goldstone bosons in the spontaneously broken phase of the theory. In this paper we develop a novel symmetry-improved CJT formalism for consistently encoding global symmetries in a loopwise expansion. In our formalism, the extremal solutions of the fields and propagators to a loopwise truncated CJT effective action are subject to additional constraints given by the Ward Identities due to global symmetries. By considering a simple $\mathbb{O}(2)$ scalar model, we show that, unlike other methods, our approach satisfies a number of important field-theoretic properties. In particular, we find that the Goldstone boson resulting from spontaneous symmetry breaking of $\mathbb{O}(2)$ is massless and the phase transition is a *second-order* one, already in the Hartree–Fock approximation. After taking the sunset diagrams into account, we show how our approach properly describes the threshold properties of the massless Goldstone boson and the Higgs particle in the loops. Finally, assuming minimal modifications to the Hartree–Fock approximated CJT effective action, we calculate the corresponding symmetry-improved CJT effective potential and discuss the conditions for its uniqueness for scalar-field values away from its minimum.

Keywords: CJT formalism; Goldstone theorem; Phase transition; Effective potential

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0550-3213/$ – see front matter
http://dx.doi.org/10.1016/j.nuclphysb.2013.06.004

1. Introduction

In quantum thermal field theory, finite-order perturbative expansions break down at high temperatures [1] and one needs to devise resummation methods to deal with this problem [2]. A natural framework to address such problems of resummation is the formalism introduced by Cornwall, Jackiw and Tomboulis (CJT) [3]. In its simplest version, the so-called Two-Particle-Irreducible (2PI) effective action is expressed not only in terms of the background field, but also in terms of the respective dressed propagator. To all orders in perturbation theory, the 2PI effective action is formally equivalent to the standard One-Particle-Irreducible (1PI) effective action. For practical purposes, however, one is compelled to consider truncations to the 2PI effective action, in terms of a loopwise diagrammatic expansion. At any given order of this loopwise expansion, the 2PI effective action contains an infinite set of diagrams induced by partially resummed propagators. There is an extensive literature related to the CJT formalism, within the context of thermal field theory [4,5], with the aim to address various problems of equilibrium and non-equilibrium dynamics (e.g. see [6] and references therein).

One major difficulty in a loopwise expansion of the CJT effective action is that global symmetries are not exactly maintained at a given loop order of the expansion, but they get distorted by higher-order effects. This problem should be contrasted with the loopwise expansion of the usual 1PI effective action, which does not suffer from this pathology and respects all global and local symmetries order by order in perturbation theory. Therefore, one important criterion for a consistent truncation of the CJT effective action is that Ward Identities (WIs) associated with global and local symmetries of the theory are satisfied by the extremal solutions of the background fields and their respective propagators. In particular, for the case of global $\mathbb{O}(N)$ symmetries that we will be studying here, a naive truncation of the CJT effective action violates the Goldstone theorem [7,8] by higher-order terms, giving rise to a massive Goldstone boson in the Spontaneous Symmetry Breaking (SSB) phase of the theory [9–12]. Thus far, several studies have been presented in the literature, attempting to provide a satisfactory solution to this problem [11–20].

It is known that a scalar $\mathbb{O}(N)$ theory has a second-order thermal phase transition. This fact is expected on general grounds, since a four-dimensional thermal field theory at high temperatures can be effectively described by a three-dimensional field theory, which is known to possess a second-order phase transition. Also, a rigorous renormalization-group analysis [21] supports this result. In the first non-trivial truncation of the 2PI effective action, the Hartree–Fock (HF) approximation [22–24], one explicitly finds [10–12] that the Goldstone boson is massive and the phase transition is first order. Only in the large-N limit of the HF approximation, a consistent prediction is obtained [11,12], where the Goldstone boson is massless and the phase transition is second order.

A first attempt in restoring the Goldstone theorem within the CJT formalism was to add a phenomenological term to the 2PI effective action [13,14]. In the HF approximation, such an approach has as major drawback. It predicts a sequence of two second-order phase transitions, including an unnatural symmetric phase of the theory, in which the masses of the Goldstone and Higgs particles are different, even though the vacuum expectation value (VEV) of the background field vanishes. Another approach employs a Two-Point-Particle-Irreducible effective action [15], but the Goldstone boson turns out to be massive at the next-to-leading order in a $1/N$ expansion.

A satisfactory field-theoretic solution must ensure that the dynamical or the threshold properties of the Goldstone and Higgs particles are properly accounted for. In particular, the Goldstone boson should consistently appear as a massless particle within quantum loops. As a consequence, in the $\mathbb{O}(N)$ model, the Higgs particle will always decay into two massless Goldstone bosons,