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G_4-flux, chiral matter and singularity resolution in F-theory compactifications

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Abstract

We construct a set of chirality inducing G_4-fluxes in global F-theory compactifications on Calabi–Yau four-folds. Special emphasis is put on models with gauge group $SU(5) \times U(1)_X$ relevant in the context of F-theory GUT model building, which are described in terms of a $U(1)$-restricted Tate model. A G_4-flux arises in a manner completely analogous to the $U(1)_X$ gauge potential. We describe in detail the resolution by blow-up of the various singularities responsible for the $U(1)_X$ factor and the standard $SU(5)$ gauge group and match the result with techniques applied in the context of toric geometry. This provides an explicit identification of the structure of the resolved fibre over the matter curves and over the enhancement points relevant for Yukawa couplings. We compute the flux-induced chiral index both of $SU(5)$ charged matter and of $SU(5)$ singlets charged only under $U(1)_X$ localised on curves which are not contained in the $SU(5)$ locus. We furthermore discuss global consistency conditions such as D3-tadpole cancellation, D-term supersymmetry and Freed–Witten quantisation. The $U(1)_X$ gauge flux is a global extension of a class of split spectral cover bundles. It constitutes an essential ingredient in the construction of globally defined F-theory compactifications with chiral matter. We exemplify this in a three-generation $SU(5) \times U(1)_X$ model whose flux satisfies all of the above global consistency conditions. We also extend our results to chiral fluxes in models without $U(1)$ restriction.

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1. Introduction

F-theory [1] provides an elegant framework to study a very broad class of string vacua. Its power and its beauty are rooted in the geometrisation of the back-reaction of physical objects, here seven-branes of Type IIB string theory, on the ambient space. This is achieved by means of a non-trivial fibration of an auxiliary elliptic curve over the physical space–time; its complex structure represents the varying axio-dilaton sourced by the seven-branes. The holomorphic nature of the relevant geometric data – seven-branes wrap divisors of the base of the fibration upon compactification to four dimensions – makes the study of the associated string vacua amenable to techniques of algebraic geometry. These geometric methods give us insights into systems beyond the perturbative realm such as mutually non-local $[p,q]$-seven-branes. The resulting marriage between the concept of brane localised gauge degrees of freedom and the appearance of exceptional gauge groups is largely responsible for the revived recent interest, triggered by [2–6], in F-theory also from a phenomenological perspective (see [7–9] for reviews on F-theory and its recent applications).

Motivated by the prospects of local F-theory model building in the context of GUT phenomenology, a great deal of recent effort has gone into the construction of globally consistent four-dimensional F-theory vacua. From the start, it has been clear that the key to the construction of such vacua and to understanding their properties is having a handle on the singularity structure of elliptic four-folds. This is because the non-abelian gauge groups, the matter spectrum and the Yukawa interactions of a model are in one-to-one correspondence with the singularities in the fibre of the Calabi–Yau four-fold over loci of, respectively, complex co-dimension one, two and three on the base (see [10] for a description of the relevant Tate algorithm and [11,12] for more recent extensions thereof). In order to make sense of the four-dimensional effective action via dimensional reduction of the dual M-theory, discussed in detail in [13], it is necessary to work not with this singular four-fold Y_4, but rather with a resolved Calabi–Yau $\hat{Y}_4$. Mathematically, the singular points in the fibre are replaced by a collection of $\mathbb{P}^1$s whose intersection structure reproduces the Dynkin diagram of the simple group associated with the singularity. Physically, resolving this singularity corresponds to moving in the Coulomb branch of the non-abelian gauge groups in the dual M-theory. In the F-theory limit of vanishing fibre volume, the resolved space $\hat{Y}_4$ and the singular Y_4 are indistinguishable. However, it is in terms of the smooth and well-defined $\hat{Y}_4$ that all computations are performed.

1.1. Singular elliptic fibrations and their resolutions

The techniques for resolution of singular elliptic fibrations were applied to F-theory soon after its discovery, starting mainly in compactifications to six dimensions. Most notably, using the powerful tools of toric geometry, an efficient algorithm was developed to completely resolve singular Calabi–Yau three-folds that are hypersurfaces of toric spaces [14,15]. In the context of F-theory GUT model building the first complete resolutions of Calabi–Yau four-folds with $SU(5)$ gauge group, as required in the spirit of [2–5], were constructed in [16,17]. This was done likewise in the framework of toric geometry, generalising the methods of [14,15] to four-folds constructed as complete intersections of toric ambient spaces. As demonstrated in [18,19], the efficiency of the toric approach allows for a systematic construction and study of a large set of four-dimensional F-theory GUT vacua which, in particular, comprises the full four-fold associated with the base space constructed previously in [20], see also [21]. It is important to stress that the toric resolution automatically takes care not only of the co-dimension one singularities,