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Nuclear Physics B 850 [FS] (2011) 321–338
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

Experimental probes of emergent symmetries in the quantum Hall system

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Received 2 February 2011; received in revised form 21 April 2011; accepted 28 April 2011

Available online 5 May 2011

Abstract

Experiments studying renormalization group flows in the quantum Hall system provide significant evidence for the existence of an emergent holomorphic modular symmetry $\Gamma_0(2)$. We briefly review this evidence and show that, for the lowest temperatures, the experimental determination of the position of the quantum critical points agrees to the parts *per mille* level with the prediction from $\Gamma_0(2)$. We present evidence that experiments giving results that deviate substantially from the symmetry predictions are not cold enough to be in the quantum critical domain. We show how the modular symmetry extended by a non-holomorphic particle–hole duality leads to an extensive web of dualities related to those in plateau–insulator transitions, and we derive a formula relating dual pairs (B, B_d) of magnetic field strengths across any transition. The experimental data obtained for the transition studied so far is in excellent agreement with the duality relations following from this emergent symmetry, and rule out the duality rule derived from the "law of corresponding states". Comparing these generalized duality predictions with future experiments on other transitions should provide stringent tests of modular duality deep in the non-linear domain far from the quantum critical points.

Keywords: Quantum Hall effect; Emergent modular symmetry

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0550-3213/$ – see front matter
doi:10.1016/j.nuclphysb.2011.04.020

1. Introduction

The possibility that the quantum Hall (QH) system should possess a discrete non-Abelian emergent symmetry relating the complex conductivity $\sigma = \sigma_H + i\sigma_D$ in different QH phases was suggested by us some time ago [1], in order to explain the apparent "super-universality" found in transitions between both integer and fractional levels in the QH system. The structure of the group emerged from the need to incorporate dualities between dyonic charges, as well as the periodicity associated with the topology of an effective theory encoding the anyonic nature of the quasi-particles. Together this led to the holomorphic modular symmetry $\Gamma_0(2)$. The symmetry was completed to a group that we shall call Γ_{H}, through the inclusion of a particle–hole duality that we study in detail below.[1]

Γ_{H} successfully predicts the full phase diagram of the QH system, both integer and fractional, including the position of the quantum critical points governing the scaling behaviour of transitions between QH levels, as well as the scaling exponents. A physical interpretation of this symmetry, based on the interchange of quasi-particles (describing the plateaux) and vortices (describing the QH insulator), was developed in Ref. [2].

In a related approach [3] based on an effective field theory incorporating "charge–flux transformations", a set of rules relating QH states at different filling factors, known collectively as "the law of corresponding states", was constructed. This also determines the topology of the phase diagram, but neither the location of quantum critical points, nor the geometry of renormalization group (RG) flows in the complex conductivity plane was obtained. It also contains a kind of duality [4,6], but one that differs substantially from the modular duality contained in the symmetry Γ_{H}.

We first discuss some of the substantial experimental evidence for the modular symmetry Γ_{H}. Not only does the symmetry describe the properties of the observed integer and fractional QH plateaux, it also determines the temperature driven RG flow of the system [8]. As briefly reviewed below, the position of the unstable fixed points and the RG trajectories are in good agreement with many experiments, particularly those involving the lowest temperatures, and so are the critical exponents. Indeed the one experiment that has been conducted at extremely low temperatures, an order of magnitude lower than previously available [7], confirms the fixed point structure predicted by the symmetry to very high accuracy. We also discuss the experimental cases that are in apparent disagreement with the Γ_{H} symmetry and provide evidence that this is because the experiments are not cold enough, i.e., that they are not probing the scaling domain.

In addition to the temperature driven flows there is a significant body of experimental work studying the magnetic field driven transitions between the QH insulator (QHI) and both integer (IQH) and fractional (FQH) plateaux. In most cases this flow is consistent with the flow predicted by the holomorphic emergent symmetry; the cases in apparent disagreement are again conducted at relatively high temperature and it is very likely they are not probing the scaling domain.

The plateau–insulator experiments also provide a sensitive test of the nature of the duality present in the quantum Hall system. The first direct experimental evidence for a duality symmetry in the QH system was obtained over a decade ago [4], but there does not appear to have been any experimental follow-up of this important discovery. The data were immediately interpreted as evidence in favour of a new "charge–flux duality" [4,6]. In subsequent sections we revisit the interpretation of this pioneering experiment, and clarify the distinction between our approach

[1] The technical definition of the quantum Hall symmetry Γ_{H} involves the group of modular automorphisms, discussed in the next section.