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Nuclear Physics B 850 [PM] (2011) 214–231
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

Hyperconifold transitions, mirror symmetry, and string theory

Rhys Davies

Mathematical Institute, University of Oxford, 24-29 St Giles, Oxford OX1 3LB, UK

Received 16 March 2011; accepted 18 April 2011

Available online 22 April 2011

Abstract

Multiply-connected Calabi–Yau threefolds are of particular interest for both string theorists and mathematicians. Recently it was pointed out that one of the generic degenerations of these spaces (occurring at codimension one in moduli space) is an isolated singularity which is a finite cyclic quotient of the conifold; these were called hyperconifolds. It was also shown that if the order of the quotient group is even, such singular varieties have projective crepant resolutions, which are therefore smooth Calabi–Yau manifolds. The resulting topological transitions were called hyperconifold transitions, and change the fundamental group as well as the Hodge numbers. Here Batyrev's construction of Calabi–Yau hypersurfaces in toric fourfolds is used to demonstrate that certain compact examples containing the remaining hyperconifolds — the $\mathbb{Z}_3$ and $\mathbb{Z}_5$ cases — also have Calabi–Yau resolutions. The mirrors of the resulting transitions are studied and it is found, surprisingly, that they are ordinary conifold transitions. These are the first examples of conifold transitions with mirrors which are more exotic extremal transitions. The new hyperconifold transitions are also used to construct a small number of new Calabi–Yau manifolds, with small Hodge numbers and fundamental group $\mathbb{Z}_3$ or $\mathbb{Z}_5$. Finally, it is demonstrated that a hyperconifold is a physically sensible background in Type IIB string theory. In analogy to the conifold case, non-perturbative dynamics smooth the physical moduli space, such that hyperconifold transitions correspond to non-singular processes in the full theory.

Keywords: String theory; Calabi–Yau manifolds; Mirror symmetry

E-mail address: daviesr@maths.ox.ac.uk.

0550-3213/$ – see front matter
doi:10.1016/j.nuclphysb.2011.04.010

1. Introduction and discussion

This paper is a follow-up to [1], in which a class of threefold singularities and associated topological transitions were studied. These are isolated Calabi–Yau threefold singularities which are quotients of the conifold by a finite cyclic group $\mathbb{Z}_N$; such a singularity was named a $\mathbb{Z}_N$-hyperconifold. They occur naturally in singular Calabi–Yau varieties which are limits of families of smooth multiply-connected spaces, when the generically-free group action on the covering space develops a fixed point.

It was shown in [1] that any projective variety with a $\mathbb{Z}_{2M}$-hyperconifold singularity has a projective crepant resolution, establishing the existence of hyperconifold *transitions* between smooth compact Calabi–Yau threefolds. The analysis was not sufficient to demonstrate the existence of the remaining cases — the $\mathbb{Z}_3$- and $\mathbb{Z}_5$-hyperconifold transitions — as the local resolution process did not guarantee that the resolved manifold was projective (and hence Kähler). Like the more familiar conifold transitions, hyperconifold transitions change the Hodge numbers; for a $\mathbb{Z}_N$-hyperconifold transition, the change is

$$\delta\big(h^{1,1}, h^{2,1}\big)_{\mathbb{Z}_N} = (N-1, -1). \tag{1}$$

A novel feature is that the fundamental group can also change.

The present work has several objectives. We work mainly within the class of Calabi–Yau hypersurfaces in toric fourfolds, first described systematically by Batyrev [2] and then enumerated by Kreuzer and Skarke [3]. The formalism is reviewed in Section 2, and then used in Section 3 to demonstrate that $\mathbb{Z}_3$- and $\mathbb{Z}_5$-hyperconifold transitions do connect compact Calabi–Yau manifolds. Perhaps more interestingly, it can also be used to study the mirror processes to these transitions, which turn out to be ordinary conifold transitions. They therefore provide a counterexample to an old conjecture of Morrison [4] that the mirror of a conifold transition is another conifold transition. The examples herein show that, while this is a very tempting conjecture, it is not true in general. They also motivate a modest conjecture, that the mirror process to any $\mathbb{Z}_N$-hyperconifold transition is a conifold transition in which the intermediate variety has N nodes. It is probably possible to use the local techniques of [5,6] to prove this [7].

The mirror conifold transitions have another interesting feature. Batyrev and Kreuzer showed that within the class of Calabi–Yau hypersurfaces in toric fourfolds, mirror symmetry exchanges the fundamental group (which in these cases can only be $\mathbb{Z}_2$, $\mathbb{Z}_3$ or $\mathbb{Z}_5$) with the Brauer group, which is the torsion part of $H^3(X,\mathbb{Z})$ [8]. Since the hyperconifold transitions studied here destroy the fundamental group, their mirror conifold transitions should destroy the Brauer group. This is not a new phenomenon (see for example [9]), but here mirror symmetry gives a clear reason for it to occur.

Once we know that hyperconifold transitions exist, we can use them to try to construct new Calabi–Yau manifolds. This was mentioned in [1], but no explicit examples were given. In Sections 3.1.3 and 3.2.1, we use the new results of this paper to construct some previously unknown Calabi–Yau manifolds via $\mathbb{Z}_3$- and $\mathbb{Z}_5$-hyperconifold transitions.

If two Calabi–Yau manifolds are mathematically connected by a topological transition, we might ask whether the corresponding physical theories, obtained by compactifying string theory on these spaces, are also smoothly connected. It is shown in Section 4 that the physical moduli space, at least in Type IIB string theory, is perfectly smooth through a point corresponding to a hyperconifold transition. The story is very similar to that of a conifold transition, worked out in [10]. The results of [1] and the present paper therefore have significant implications for the connectedness of the moduli space of Calabi–Yau threefolds, and the associated string vacua.