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Lagrangian formulations of self-dual gauge theories in diverse dimensions

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Abstract

In this work, we study Lagrangian formulations for self-dual gauge theories, also known as chiral n-form gauge theories, for $n = 2p$ in $D = 4p + 2$ dimensional spacetime. Motivated by a recent formulation of M5-branes derived from the BLG model, we generalize the earlier Lagrangian formulation based on a decomposition of spacetime into $(D - 1)$ dimensions plus a special dimension, to construct Lagrangian formulations based on a generic decomposition of spacetime into D' and $D'' = D - D'$ dimensions. Although the Lorentz symmetry is not manifest, we prove that the action is invariant under modified Lorentz transformations.

1. Introduction

Self-dual gauge theories, or chiral theories, are full of both physical and mathematical interests. The goal of this work is to provide new Lagrangian formulations of self-dual gauge theories for spacetime dimensions equal to 2 modulo 4, i.e., $D = 4p + 2$, for $p = 0, 1, 2, 3, \ldots$. This includes chiral bosons in 2D, self-dual 3-form gauge field for M5-brane in M theory and self-dual 5-form gauge field in type IIB superstring theory. For simplicity we will assume that the spacetime is Minkowski space. It should be straightforward to generalize the formulation to curved spacetime with Lorentzian signature.

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Let us recall that the dual of a tensor T in D dimensions is defined as

$$\tilde{T}_{\mu_1\ldots\mu_k} \equiv \frac{1}{(D-k)!}\epsilon_{\mu_1\ldots\mu_D}T^{\mu_{k+1}\ldots\mu_D}. \tag{1}$$

The self-duality condition of a gauge field is

$$\mathcal{F} \equiv F - \tilde{F} = 0. \tag{2}$$

Apparently the field strength must be a $D/2$-form, and D must be even. Self-duality conditions are not consistent in $D = 4p$ ($p = 1, 2, 3, \ldots$) dimensional Minkowski space, and they will not be considered in this work.

It is well known that, without auxiliary fields, a manifestly Lorentz invariant action cannot be found for self-dual gauge theory. In the literature on self-dual theories [1,2], a special (but arbitrary) direction has to be singled out to write down a Lagrangian. Recently, in the study of M theory, a new Lagrangian formulation of M5-brane in large C-field background [3–5], which is a self-dual gauge field theory in 6 dimensions, was derived from the BLG model [6,7] for multiple M2-branes. In this formulation, the 6 coordinates of the base space are divided into two sets of 3 coordinates $\{x^a\}_{a=1}^3$ and $\{x^{\dot{a}}\}_{\dot{a}=1}^3$, in contrast with the old formulation of M5-branes [8], which uses a decomposition of base space coordinates into two sets of 1 and 5 coordinates. While both decompositions $6 = 1 + 5$ and $6 = 3 + 3$ admit Lagrangian formulations of self-dual theories, the natural question is: does there exist a formulation corresponding to the decomposition $6 = 2 + 4$? More generally, for a self-dual gauge theory in $D = 4p + 2$ dimensions (so the gauge field strength is a $(2p + 1)$-form), can we find a Lagrangian formulation for all possible decompositions $D = D' + D''$?

The answer to the question above is yes. In the following, we provide new Lagrangian formulations for arbitrary spacetime decompositions. The action is given in Section 2, its gauge symmetry in Section 3. The proof that the theory is a theory of self-dual gauge fields is given separately for three classes of decompositions: (i) $D' = D''$ (Section 4), (ii) $D'' > D' =$ odd (Section 5) and (iii) $D'' > D' =$ even (Section 6). We show in Section 7 that although the action is no longer manifestly invariant under those Lorentz transformations which mix the two sets of coordinates in the decomposition, the action is invariant under certain modified Lorentz transformation laws. In Section 8, we give the interaction term in the action to describe the coupling of the gauge field to a charged $(2p - 1)$-brane. Explicit examples for $(D', D'') = (1, 1)$, $(1, 5)$, $(2, 4)$, $(3, 3)$ are given in Section 9. We point out the relationship between our result and the holographic action of Belov and Moore [9] in Section 10 and our conclusion will be given in Section 11.

2. Action

We decompose the D-dimensional Minkowski spacetime as a product space $\mathcal{M}^D = \mathcal{M}_1^{D'} \times \mathcal{M}_2^{D''}$. Correspondingly, the spacetime coordinates $\{x^\mu \mid \mu = 1, \ldots, D\}$ are divided into two sets

$$\mathcal{M}_1^{D'}\colon \{x^a \mid a = 1, \ldots, D'\} \quad \text{and} \quad \mathcal{M}_2^{D''}\colon \{x^{\dot{a}} \mid \dot{a} = 1, \ldots, D''\}. \tag{3}$$

We assume that $D' \leqslant D''$, so that $1 \leqslant D' \leqslant D/2$. This is just a convention except that when D' is even, the signature of $\mathcal{M}_1^{D'}$ must be Lorentzian, and the signature of $\mathcal{M}_2^{D''}$ Euclidean. The Lorentzian signature of spacetime can be either $(+ - \cdots -)$ or $(- + \cdots +)$. The expressions below are valid for both conventions.