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Four-loop massless propagators: A numerical evaluation of all master integrals

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Abstract

We present numerical results which are needed to evaluate all non-trivial master integrals for four-loop massless propagators, confirming the recent analytic results of Baikov and Chetyrkin (2010) [1] and evaluating an extra order in ε expansion for each master integral.

Keywords: Feynman diagrams; Massless propagators; Master integrals; Sector decomposition; Numerical evaluation

1. Introduction

Sector decomposition in its practical aspect is a constructive method used to evaluate Feynman integrals numerically. The goal of sector decomposition is to decompose the initial integration domain into appropriate subdomains (sectors) and introduce, in each sector, new variables in such a way that the integrand factorizes, i.e. becomes equal to a monomial in new variables times a non-singular function.

Originally it was used as a tool for analyzing the convergence and proving theorems on renormalization and asymptotic expansions of Feynman integrals [2–6]. After a pioneering work [7] sector decomposition has become an efficient tool for numerical evaluating Feynman integrals (see Ref. [8] for a recent review). At present, there are two public codes performing the sector decomposition [9] and [10].

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The latter one was named FIESTA which stands for "Feynman Integral Evaluation by a Sector decompositiOn Approach". Last year FIESTA has been greatly improved in various aspects [11]. The code is capable of evaluating many classes of integrals that one would not be able to evaluate with the original FIESTA 1. Moreover the code can now be applied to solve the problem of obtaining asymptotic expansions of Feynman integrals in various limits of momenta and masses and to find a list of all poles of an integral in space–time dimension d. During the last year FIESTA was widely used, some of application are listed in [12].

In the current paper we present numerical results for *master integrals* (MI's) for four-loop massless propagators which are relevant for many important physical applications, like the calculation of the total cross-section of e^+e^- annihilation into hadrons, the Higgs decay rate into hadrons, the semihadronic decay rate of the τ lepton and the running of the fine structure coupling constant (see [13,14] for details). We confirm numerically the recent analytic results of work [1] and evaluate an extra order in epsilon expansion for each MI.

2. Theoretical background and software structure

FIESTA calculates Feynman integrals with the sector decomposition approach. It is based on the α-representation of Feynman integrals. After performing Dirac and Lorentz algebra one is left with a scalar dimensionally regularized Feynman integral [15]

$$F(a_1, \ldots, a_n) = \int \cdots \int \frac{\mathrm{d}^d k_1 \ldots \mathrm{d}^d k_l}{E_1^{a_1} \ldots E_n^{a_n}}, \tag{1}$$

where $d = 4 - 2\varepsilon$ is the space–time dimension, a_n are indices, l is the number of loops and $1/E_n$ are propagators. We work in Minkowski space where the standard propagators are the form $1/(m^2 - p^2 - i0)$. Other propagators are permitted, for example, $1/(v \cdot k \pm i0)$ may appear in diagrams contributing to static quark potentials or in HQET[1] where v is the quark velocity (see, e.g. [16]). Substituting

$$\frac{1}{E_i^{a_i}} = \frac{e^{ai\pi/2}}{\Gamma(a)} \int_0^\infty \mathrm{d}\alpha\, \alpha^{a_i - 1} e^{-iE_i \alpha}, \tag{2}$$

changing the integration order, performing the integration over loop momenta, replacing α_i with $x_i \eta$ and integrating over η one arrives at the following formula (see, e.g. [17]):

$$F(a_1, \ldots, a_n) = \frac{\Gamma(A - ld/2)}{\prod_{j=1}^n \Gamma(a_j)} \int_{x_j \geqslant 0} dx_i \ldots dx_n \delta\left(1 - \sum_{i=1}^n x_i\right) \left(\prod_{j=1}^n x_j^{a_j - 1}\right) \frac{U^{A-(l+1)d/2}}{F^{A-ld/2}}, \tag{3}$$

where $A = \sum_{i=1}^n a_n$ and U and F are constructively defined polynomials of x_i. The formula (3) has no sense if some of the indices are non-positive integers, so in case of those the integration is performed according to the rule

$$\int_0^\infty dx \frac{x^{(a-1)}}{\Gamma(a)} f(x) = f^{(n)}(0)$$

where a is a non-positive integer.

[1] **H**eavy-**Q**uark **E**ffective **T**heory.