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Longitudinal and transverse spectral functions in the three-dimensional $O(4)$ model

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Abstract

We have performed a high statistics simulation of the $O(4)$ model on a three-dimensional lattice of linear extension $L = 120$ for small external fields H. Using the maximum entropy method we analyze the longitudinal and transverse plane spin correlation functions for $T < T_c$ and $T \geqslant T_c$. In the transverse case we find for all T and H a *single* sharp peak in the spectral function, whose position defines the transverse mass m_T, the correlator is that of a free particle with mass m_T. In the longitudinal case we find in the very high temperature region also a single sharp peak in the spectrum. On approaching the critical point from above the peak broadens somewhat and at T_c its position m_L is at $2m_T$ for all our H-values. Below T_c we find still a significant peak at $\omega = 2m_T$ and at higher ω-values a continuum of states with several smaller peaks with decreasing heights. This finding is in accord with a relation of Patashinskii and Pokrovskii between the longitudinal and the transverse correlation functions. We test this relation and its range of applicability in the following. As a by-product we calculate critical exponents and amplitudes and confirm our former results.

Keywords: $O(4)$ model; Correlation functions; Maximum entropy method; Spectral function

1. Introduction

$O(N)$ spin models with $N > 1$ possess two types of two-point correlation functions, one for the transverse and one for the longitudinal spin component, defined relative to the direction of the external field $\vec{H}$. Correspondingly, there exist also two distinct susceptibilities and correlation

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lengths. The difference and interplay between the two sets of observables is of interest not only for the critical behaviour near to T_c, the critical temperature, but as well for the study of the singularities induced by the existence of the massless Goldstone modes in these $O(N)$ spin models for dimension $2 < d \leqslant 4$ and all $T < T_c$ [1,2].

In a previous paper [3] we studied the correlation lengths which govern the exponential decay of the two correlation functions in the three-dimensional $O(4)$ model on lattices of linear extensions $L = 48$–120. We were able to confirm the predicted singular Goldstone behaviour [4,5] of the transverse correlation length near the coexistence line, $T < T_c$, $H = 0$ and to determine the scaling function of ξ_T. For the longitudinal correlation length the situation was different. In the high temperature region the scaling function could be calculated, in the low temperature phase, $T < T_c$, we were however unable to reliably estimate ξ_L, the data were not even scaling. This was ascribed to a spectrum of higher states which contribute to the longitudinal correlators below T_c. In this paper we want to calculate the spectral functions of the two correlators and to actually find these states above the ground states $m_{T,L} = 1/\xi_{T,L}$. Here, not only the spectrum of the longitudinal correlator is of interest, but as well the transverse spectrum. Spin-wave theory assumes, that long-wavelength transverse fluctuations dominate for small fields in thermal equilibrium below T_c and that these fluctuations are describable by the Gaussian model or free field functional. This assumption can be tested readily by comparing the transverse correlator to the known Gaussian form. Of course, no higher state should then appear in the spectrum.

The resulting dependence on H of the lowest states m_T and m_L at fixed T can be utilized to test critical behaviour and the effects of massless Goldstone modes. At the critical point we had already found in Ref. [3] that $m_L = 2m_T$, like for $O(2)$ [6]. The equation between the two masses or correlation lengths follows from a relation between the transverse and longitudinal correlation functions [5,4] which should be valid for small H in the whole low temperature phase. We shall discuss and test this relation explicitly for its range of applicability. In order to achieve these goals, we simulate the $O(4)$-invariant nonlinear σ-model with high statistics on a lattice with linear extension $L = 120$. We have chosen this specific model for several reasons: first because we want to clarify the open points raised in Ref. [3], second because in contrast to the corresponding $O(2)$-model corrections to scaling are negligible here, and third because the model is of relevance for quantum chromodynamics (QCD) with two degenerate light-quark flavours at finite temperature, since its phase transition is supposed to belong to the same universality class as the chiral transition of QCD [7–9].

The $O(N)$-model which we study here is defined by the Hamiltonian

$$\beta\mathcal{H} = -J \sum_{\langle\vec{x},\vec{y}\rangle} \vec{\phi}(\vec{x}) \cdot \vec{\phi}(\vec{y}) - \vec{H} \sum_{\vec{x}} \vec{\phi}(\vec{x}), \tag{1}$$

where $\vec{x}$ and $\vec{y}$ are nearest-neighbour sites on a three-dimensional hypercubic lattice with periodic boundary conditions, and $\vec{\phi}(\vec{x})$ is a N-component spin vector at site $\vec{x}$ with length 1. It is convenient to give the following formulas still for general N, though we use in our simulations $N = 4$. The magnetization vector $\vec{M}$ is the expectation value of the lattice average $\vec{\phi}$ of the local spins

$$\vec{M} = \langle\vec{\phi}\rangle, \quad \text{with } \vec{\phi} = \frac{1}{V} \sum_{\vec{x}} \vec{\phi}(\vec{x}). \tag{2}$$

Here, $V = L^3$ and L is the number of lattice points per direction, the lattice spacing a is fixed to 1. Due to the invariance of $\mathcal{H}_0$, the $\vec{H}$-independent part of $\mathcal{H}$, under $O(N)$-rotations of $\vec{\phi}(\vec{x})$, the magnetization vector aligns with $\vec{H}$