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Superconformal indices for $\mathcal{N}=1$ theories with multiple duals

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Abstract

Following a recent work of Dolan and Osborn, we consider superconformal indices of four-dimensional $\mathcal{N}=1$ supersymmetric field theories related by an electric–magnetic duality with the $SP(2N)$ gauge group and *fixed rank* flavour groups. For the $SP(2)$ (or $SU(2)$) case with 8 flavours, the electric theory has index described by an elliptic analogue of the Gauss hypergeometric function constructed earlier by the first author. Using the E_7-root system Weyl group transformations for this function, we build a number of dual magnetic theories. One of them was originally discovered by Seiberg, the second model was built by Intriligator and Pouliot, the third one was found by Csáki et al. We argue that there should be in total 72 theories dual to each other through the action of the coset group $W(E_7)/S_8$. For the general $SP(2N)$, $N>1$, gauge group, a similar multiple duality takes place for slightly more complicated flavour symmetry groups. Superconformal indices of the corresponding theories coincide due to the Rains identity for a multidimensional elliptic hypergeometric integral associated with the BC_N-root system.

1. Introduction

One of the more important recent achievements of mathematical physics consists of the discovery of elliptic hypergeometric functions — a new class of special functions of hypergeometric type (see [1] for a survey of the corresponding results and relevant literature). These functions have found applications in the theory of Yang–Baxter equation, integrable discrete time chains,

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elliptic Calogero–Sutherland type models and so on. Although connection with the classical root systems has been explicitly traced in the structure of many elliptic hypergeometric functions, their group theoretical interpretation remained largely obscure.

In recent papers Römelsberger [2] and Kinney et al. [3] have described topological indices for four-dimensional supersymmetric conformal field theories. As suggested in [2], superconformal indices of the $\mathcal{N}=1$ models related by Seiberg duality [4,5] should coincide as a result of some complicated group theoretical identities. Following Römelsberger's ideas, Dolan and Osborn [6] have connected superconformal indices of a number of $\mathcal{N}=1$ supersymmetric field theories with specific elliptic hypergeometric integrals. Corresponding dual theories have the same indices due to nontrivial identities for these integrals [1].

For example, in [7] the first author has discovered the elliptic beta integral opening the door to a new class of computable integrals. It is described by the following exact integration formula:

$$\frac{(p;p)_\infty(q;q)_\infty}{2}\int_{\mathbb{T}}\frac{\prod_{j=1}^6\Gamma(t_jz^{\pm1};p,q)}{\Gamma(z^{\pm2};p,q)}\frac{dz}{2\pi iz}=\prod_{1\leqslant j<k\leqslant 6}\Gamma(t_jt_k;p,q), \tag{1}$$

where six complex parameters t_j, $j=1,\dots,6$, and two base variables p and q satisfy the inequalities $|p|,|q|,|t_j|<1$ and the balancing condition

$$\prod_{j=1}^6 t_j=pq.$$

Here $\mathbb{T}$ denotes the unit circle with positive orientation and

$$\Gamma(z;p,q):=\prod_{j,k=0}^\infty\frac{1-z^{-1}p^{j+1}q^{k+1}}{1-zp^jq^k}$$

is the elliptic gamma function. In (1) and below we denote $(t;q)_\infty:=\prod_{k=0}^\infty(1-tq^k)$ and use the conventions

$$\Gamma\big(tz^{\pm1};p,q\big):=\Gamma(tz;p,q)\Gamma\big(tz^{-1};p,q\big),$$
$$\Gamma\big(z^{\pm2};p,q\big):=\Gamma\big(z^2;p,q\big)\Gamma\big(z^{-2};p,q\big),$$
$$\Gamma\big(tz^{\pm1}w^{\pm1};p,q\big):=\Gamma(tzw;p,q)\Gamma\big(tzw^{-1};p,q\big)\Gamma\big(tz^{-1}w;p,q\big)\Gamma\big(tz^{-1}w^{-1};p,q\big).$$

As shown by Dolan and Osborn [6], the left-hand side of formula (1) describes the superconformal index of the "electric" theory with $SU(2)$ gauge group and quark superfields in the fundamental representation of the $SU(6)$ flavour group. The "magnetic" dual theory, suggested by Seiberg in [4], does not have gauge degrees of freedom; the matter sector contains meson superfields in 15-dimensional antisymmetric $SU(6)$-tensor representation of the second rank; and its superconformal index is described by the right-hand side of relation (1). This duality provides the simplest example of the so-called s-confining theories.

Seiberg duality is a fundamental concept of the modern quantum field theory [4,5,8–16]. Corresponding models contain particular sets of fields transforming as representations of the group $G_{\rm st}\times G\times F$, where $G_{\rm st}=SU(2,2|1)$ is the space–time superconformal symmetry group (containing the R-symmetry subgroup $U(1)_R$ rotating supercharges), G is the local gauge invariance group, and F is the global flavour symmetry group. Conditionally, electric theories are considered as manifestations of a unique complicated "stringy" dynamics in the weak coupling regime.