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Formula for fixed point resolution matrix of permutation orbifolds

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Abstract

We find a formula for the resolution of fixed points in extensions of permutation orbifold conformal field theories by its (half-)integer spin simple currents. We show that the formula gives a unitary and modular invariant S matrix.

Keywords: Conformal field theory; String theory; Permutation orbifolds

1. Introduction

In a series of recent papers [1,2] we have started to study the problem of resolving the fixed points [3–5] in simple current [6–9] extensions of permutation orbifold [10,11] conformal field theories[1] [12]. The aim of this paper is to give a general solution to this problem, going much beyond the specific examples discussed previously.

Simple currents J are special fields: those with simple fusion rules with any other field in the theory. They are very important ingredients of a CFT, since they allow to modify the theory in a well-controlled way by projecting out some fields and re-organizing the remaining into new ones. In practice, what one does is: compute the monodromy charge of any field i with respect to

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[1] We will implicitly assume rational conformal field theories.

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(w.r.t.) J, $Q_J(i)$; project out those fields which have non-integer monodromy charge; organize the surviving fields into orbits under J.

In this paper we will mostly look at order-two simple currents, i.e. those with $J^2 = 1$, for which the J-orbits can have length equal to one or two at most. The normal (and easy to handle) fields are those with length two: $(i, J \cdot i)$. More special are those orbits with length one: $J \cdot f = f$. A field f satisfying this property is called a *fixed point* of the current J. Fixed points can arise only when the current J has integer or half-integer spin, given by its weight h_J. In the extension, they give rise to more than one field, whose number is equal to the order of the current. In this paper then every fixed points will split into two fields in the extended theory.

In any (rational) CFT, two of the most important objects are the modular S and T matrices. T is a diagonal matrix of phases and contains information about the weights of the fields in the theory; S is symmetric and unitary and allows to compute the fusion coefficients (conceptually analogous to the Clebsch–Gordan series) between two representations via the Verlinde formula [13]. Without fixed points, there is no difficulty in deriving the S matrix of the extended theory, sometimes denoted by $\tilde{S}$, from the S matrix of the unextended CFT. On the contrary, when fixed points are present, not only the extended S matrix is problematic to derive but also it is affected by an intrinsic ambiguity related to the freedom that we have in choosing the order of the splitted fields coming from the same fixed point. This issue has already been addressed in the past [3] and the outcome was that we can write the matrix $\tilde{S}$ in terms of a set of matrices S^J, one for each simple current J, and hence the problem of resolving the fixed points is re-formulated as the problem of determining those S^J matrices.

Using the formalism developed in [3], we can trade our ignorance about $\tilde{S}$ with a set of matrices S^J, one for every simple current J, according to the formula

$$\tilde{S}_{(a,i)(b,j)} = \frac{|G|}{\sqrt{|U_a||S_a||U_b||S_b|}} \sum_{J \in G} \Psi_i(J) S^J_{ab} \Psi_j(J)^\star. \tag{1.1}$$

These S^J_{ab}'s are non-zero only if both a and b are fixed points. This equation can be viewed as a Fourier transform and the S^J's as Fourier coefficients of $\tilde{S}$. The prefactor is a group theoretical factor acting as a normalization and the $\Psi_i(J)$'s are the group characters acting as phases. As conjectured in [3] and proved in [14], the S^J matrices describe the modular transformation properties of the one-point function on the torus with the insertion of the simple current $J(z)$. Unitarity and modular invariance of $\tilde{S}$ implies unitarity and modular invariance of the S^J's [3]:

$$S^J \cdot \left(S^J\right)^\dagger = 1, \qquad \left(S^J \cdot T^J\right)^3 = \left(S^J\right)^2. \tag{1.2}$$

In this way, the problem of finding $\tilde{S}$ is equivalent to the problem of finding the set of matrices S^J.

The unextended theory that we can consider before the extension can be any CFT $\mathcal{A}$. It can be also a tensor product of different CFT's, $\mathcal{A}_1 \otimes \cdots \otimes \mathcal{A}_n$, or even a coset theory of the form G/H. All these cases have already been considered in the past and their S^J matrices are known by now. In fact, these matrices have been found for all WZW models [3,15], their simple current extensions [4] and for coset conformal field theories [5]. In this paper we will consider the permutation orbifold as the unextended CFT, for which the S^J matrices are in general not known yet. We restrict ourselves to the case of $\mathbb{Z}_2$ orbifolds [10,11], where we mod out by the cyclic permutation that exchanges the two factors:

$$\mathcal{A}_{\text{perm}} \equiv \mathcal{A} \times \mathcal{A}/\mathbb{Z}_2. \tag{1.3}$$

Larger orbifolds would be possible [16,17], but they are much more involved and we will not treat them here.