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Differential operator realizations of superalgebras and free field representations of corresponding current algebras

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Abstract

Based on the particular orderings introduced for the positive roots of finite-dimensional basic Lie superalgebras, we construct the explicit differential operator representations of the $osp(2r|2n)$ and $osp(2r+1|2n)$ superalgebras and the explicit free field realizations of the corresponding current superalgebras $osp(2r|2n)_k$ and $osp(2r+1|2n)_k$ at an arbitrary level k. The free field representations of the corresponding energy–momentum tensors and screening currents of the first kind are also presented.
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1. Introduction

The interest in two-dimensional non-linear σ-models with supergroups or their cosets as target spaces has grown drastically over the last ten years because of their applications ranging from string theory [1,2] and logarithmic conformal field theories (CFTs) [3,4] (for a review, see e.g. [5,6], and references therein) to modern condensed matter physics [7–14]. The Wess–Zumino–Novikov–Witten (WZNW) models associated with supergroups stand out as an important class

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of such σ-models. This is due to the fact that, besides their own importance, the WZNW models are also the "building blocks" for other coset models which can be obtained by gauging or coset constructions [15–18]. In these models, current or affine (super)algebras [19] are the underlying symmetry algebras and are relevant to integrability of the model.

In contrast to the bosonic versions, the WZNW models on supergroups are far from being *understood* ([20] and references therein), although some progress has been made [21] recently for the models related to type I supergroups [22]. This is largely due to technical reasons (such as indecomposability of the operator product expansion (OPE) [23,24], appearance of logarithms in correlation functions and continuous modular transformations of the irreducible characters [25]).

On the other hand, the Wakimoto free field realizations of current algebras [26] have been proved very powerful in the study of the WZNW models on bosonic groups [27–32]. Since the work of Wakimoto on the $sl(2)$ current algebra, much effort has been made to obtain similar results for the general case [33–39]. In these constructions, the explicit differential operator realizations of the corresponding finite-dimensional (super)algebras play a key role. However, explicit differential operator expressions heavily depend on the choice of local coordinate systems in the so-called big cell $\mathcal{U}$ [40]. Thus it is at least very involved, if not impossible, to obtain explicit differential operator expressions for higher-rank (super)algebras in the usual coordinate systems [36–39,41–43]. Recently it was shown in [44–46] that there exists a certain coordinate system in $\mathcal{U}$, which drastically simplifies the computation involved in the construction of explicit differential operator expressions for higher-rank (super)algebras. We call such a coordinate system the "good coordinate system".

This paper will show how to establish a "good coordinate system" of the big cell $\mathcal{U}$ for an arbitrary finite-dimensional basic Lie superalgebra [22]. It will be seen that the "good coordinate system" *indeed* exits and is related to a particular ordering for the positive roots of the superalgebra. Based on such an ordering of the positive roots, we construct the "good coordinate system" for the superalgebras $osp(2r|2n)$ and $osp(2r+1|2n)$ and derive their explicit differential operator representations. We then apply these differential operators to construct explicit free field representations of the $osp(2r|2n)$ and $osp(2r+1|2n)$ current algebras.

This paper is organized as follows. In Section 2, we briefly review finite-dimensional simple basic Lie superalgebras and their corresponding current algebras, which also introduces our notation and some basic ingredients. In Section 3, we introduce the particular orderings for the positive roots of the superalgebras $osp(2r|2n)$ and $osp(2r+1|2n)$. Based on the orderings, we construct the explicit differential operator representations of $osp(2r|2n)$ and $osp(2r+1|2n)$. In Section 4 we apply these differential operator expressions to construct the explicit free field realizations of the $osp(2r|2n)$ and $osp(2r+1|2n)$ currents, the energy–momentum tensors and the screening currents. Section 5 provides some discussions. In Appendix A, we give the matrix forms of the defining representations of superalgebras $osp(2r|2n)$ and $osp(2r+1|2n)$.

2. Notation and preliminaries

Let $\mathcal{G} = \mathcal{G}_{\bar{0}} + \mathcal{G}_{\bar{1}}$ be a finite-dimensional simple basic Lie superalgebra [22,47] with a $\mathbb{Z}_2$-grading: $[a] = 0$ for $a \in \mathcal{G}_{\bar{0}}$ and $[a] = 1$ for $a \in \mathcal{G}_{\bar{1}}$. For any two homogenous elements (i.e. elements with definite $\mathbb{Z}_2$-gradings) $a, b \in \mathcal{G}$, the Lie bracket is defined by

$$[a, b] = ab - (-1)^{[a][b]} ba.$$

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