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Leading logarithms in the massive $O(N)$ nonlinear sigma model

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Abstract

We review Büchler and Colangelo's result that leading divergences at any loop order can be calculated using only one-loop calculations and we provide an alternative proof. We then use this method to calculate the leading divergences of and thus the leading logarithmic corrections to the meson mass in the massive $O(N)$ nonlinear sigma model to five-loop order. We also calculate the all-loop result to leading order in the large N expansion by showing that only cactus diagrams contribute and by summing these via a generalized gap equation.

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1. Introduction

In a renormalizable theory the use of Renormalization Group Equations (RGE) is common practice. RGEs have not yet received the same attention in nonrenormalizable effective theories. This is partially due to the fact that one does not normally have the problem of evolving coupling constants through a large energy range, and partially to the fact that RGEs in nonrenormalizable theories get more complicated as one goes to higher orders. Loop corrections however can be significant [1–3] and need to be dealt with.

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Consider for example the scattering length a_0^0 in $\pi\pi$ S-wave, $I = 0$ scattering in ChPT. Close to threshold, this amplitude may be expressed in terms of the expansion parameter $(M_\pi/4\pi F_\pi)^2 \sim 0.01$. Despite the smallness of the expansion parameter, the one-loop contributions cause a 28% correction to the tree level prediction [4]. The reason for this is that beyond tree level the expressions for observables contain nonanalytic functions [5] such as $M^2 \log(\mu^2/M^2)$ which may be large if M^2 is small, even near threshold. It is only natural then to wonder about the size of higher order n contributions like $(M^2 \log(\mu^2/M^2))^n$, the so-called leading logarithms, and about the size of their coefficients.

In a renormalizable theory these coefficients are fully determined by a one-loop calculation. This is a consequence of the RGEs. The difference between a renormalizable and a nonrenormalizable theory is that in the first case the counterterms needed at any given order have the same form, while in the latter case new ones are needed at every order. Nevertheless, one can still make predictions on the leading logarithms. This fact was first pointed out by Weinberg [1] in the context of Chiral Perturbation Theory (ChPT). He showed that at two loops the coefficient of leading logarithm $(M^2 \log(\mu^2/M^2))^2$ can be determined simply by performing a one-loop calculation.

This method has since been used in ChPT [1–3] to two-loop order in $\pi\pi$ scattering [6] and in general [7]. It is no longer much used in the purely mesonic sector, since most processes are actually fully known at two-loop order as reviewed in e.g. [8].

In the last few years though, Weinberg's argument has received new attention [9–13], especially since Büchler and Colangelo were able to generalize the result to all orders [9]. They showed explicitly that one can obtain the (coefficient of the) leading logarithm at any order by simply performing one-loop calculations and that this coefficient is just a function of the lowest order coupling constants. The relevant part of their paper [9] and their algorithm to find the coefficients is described in Section 2.

In Section 2.3 we provide an alternative proof of their results which does not rely explicitly on β-functions but follows directly from the fact that all nonlocal divergences must cancel. This version of the proof has the benefit that it shows immediately that one only needs to calculate the divergent part without worrying about classifying higher-order Lagrangians and that there is a direct link between divergences and leading logarithms.

We then apply this method to the case of a *massive* $O(N)$ nonlinear sigma model and calculate the corrections to the meson mass up to five loops. Section 5 contains a detailed explanation of our calculation. Similar calculations have been performed by [12,13] who showed how to obtain the leading logarithms in the massless case by deriving a recursion relation for all possible vertices with up to four mesons. Since in the massless limit the tadpoles vanish, this allows obtain the leading logarithms in a straightforward fashion.

The authors of [10] instead calculated the two-point function up to five loops in ChPT in the chiral limit using dispersive methods.[1] Once the first five leading logarithms where known, the next step was finding an algorithm that would allow them to calculate the n-th order one and eventually to resum the series. In paper [11] they considered a *linear sigma* model and compared the correlator leading logarithms they found with those from ChPT, both in the chiral limit. They showed that it is not possible to simply use RGEs in the linear sigma model to resum the chiral logarithm series. The two scales present in the linear sigma model both generate logarithms that cannot be disentangled.

[1] In the same paper the authors also calculate the dispersive part of the three-loop pion form factor.