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The superspin approach to a disordered quantum wire in the chiral-unitary symmetry class with an arbitrary number of channels

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Abstract

We use a superspin Hamiltonian defined on an infinite-dimensional Fock space with positive definite scalar product to study localization and delocalization of noninteracting spinless quasiparticles in quasi-one-dimensional quantum wires perturbed by weak quenched disorder. Past works using this approach have considered a single chain. Here, we extend the formalism to treat a quasi-one-dimensional system: a quantum wire with an arbitrary number of channels coupled by random hopping amplitudes. The computations are carried out explicitly for the case of a chiral quasi-one-dimensional wire with broken time-reversal symmetry (chiral-unitary symmetry class). By treating the space direction along the chains as imaginary time, the effects of the disorder are encoded in the time evolution induced by a single site superspin (non-Hermitian) Hamiltonian. We obtain the density of states near the band center of an infinitely long quantum wire. Our results agree with those based on the Dorokhov–Mello–Pereyra–Kumar equation for the chiral-unitary symmetry class.

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1. Introduction

In this paper we calculate the density of states in the close vicinity of the band center of a disordered quasi-one-dimensional system belonging to the chiral unitary symmetry class for an arbitrary number N of channels by using a representation of the problem in terms of an effective zero-dimensional superspin Hamiltonian. We employ the random hopping chain with complex valued hopping integrals as a representative of the chiral unitary symmetry class, which is termed class AIII in the Altland–Zirnbauer (AZ) classification [1–4]. The chiral symmetry restricts the hopping amplitudes to connecting pairs of sites with each site belonging to a different sublattice of a bipartite lattice. Such a system has the property that its spectrum of energy eigenvalues is symmetric under a local gauge transformation that exchanges pairs of energy eigenvalues of opposite signs, whatever the realization of the random hopping amplitudes. This is the reason why disordered chiral unitary symmetric Hamiltonians exhibit anomalous behavior near zero energy. Indeed, in (quasi-)one dimension, the band center in the chiral-unitary symmetry class realizes a mobility edge separating two insulating phases.

These issues were first studied by use of a transfer matrix approach [5–7] based on the Dorokhov–Mello–Pereyra–Kumar (DMPK) equation [8]. There it was shown that the density of states near the band center depends sensitively on the parity of the number of channels N. When N is odd, the density of states exhibits a singularity at zero energy. When N is even, the density of states is controlled by random matrix theory up to multiplicative logarithmic corrections. For N odd the multi-channel problem shows behavior similar to the strictly one-dimensional case ($N = 1$), which was pioneered by Dyson [9] more than fifty years ago. Over the course of the years, the single-channel chiral unitary random hopping chain has been recast in numerous ways and investigated using a great variety of different methods [10–25].

Of direct relevance to the present paper is the method of Refs. [21–25], which maps the single channel random hopping chain onto a (super)spin-like model. In the case of a single channel, the left- and right-moving modes of the quantum wire can be effectively grouped as two separate sites of a (super)spin-like model. That is, the states of the superspin model can be viewed as a tensor product space of two "superspins". In this setting one then has to solve the problem of the two coupled superspins, analogous to the "spin addition" in quantum mechanics, albeit more complicated due to the infinite dimensionality of the superspins.

Similar techniques have also been employed to study localization in two spatial dimensions. For example, the plateau transition in the integer quantum Hall effect (symmetry class A) has been described in terms of an effective one-dimensional antiferromagnetic (super)spin chain [26–32]. In this approach, the space of states is written as a tensor product of alternating highest and lowest weight infinite dimensional irreducible representations (irreps) of a Lie (super)algebra, also loosely called "(super)spins", and the Hamiltonian is built out of generators of the same algebra. This was an important conceptual step in the sense that it provided the hope that the flow to strong coupling of the corresponding nonlinear σ model (NLSM) could be captured at the level of the (super)spin chain in a way similar as the flow from the $O(3)$-NLSM to the $SU(2)_1$ Wess–Zumino–Witten conformal field theory is captured by the Bethe Ansatz solution to the quantum spin-$1/2$ Heisenberg chain [33]. Although, this hope proved ephemeral in the context of the plateau transitions (for a review see Ref. [31]), it lead to remarkable exact results when the same approach was used in the case of the two-dimensional Bogoliubov–de-Gennes symmetry class C (spin quantum Hall effect) [34–39].

Unfortunately, this success story is the exception rather than the rule at the present. For the chiral classes AIII, BDI, and CII a mapping of the two-dimensional random hopping problem onto