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Clustering properties, Jack polynomials and unitary conformal field theories

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Abstract

Recently, Jack polynomials have been proposed as natural generalizations of $\mathbb{Z}_k$ Read–Rezayi states describing non-Abelian fractional quantum Hall systems. These polynomials are conjectured to be related to correlation functions of a class of W-conformal field theories based on the Lie algebra A_{k-1}. These theories can be considered as non-unitary solutions of a more general series of CFTs with $\mathbb{Z}_k$ symmetry, the parafermionic theories. Starting from the observation that some parafermionic theories admit unitary solutions as well, we show, by computing the corresponding correlation functions, that these theories provide trial wavefunctions which satisfy the same clustering properties as the non-unitary ones. We show explicitly that, although the wavefunctions constructed by unitary CFTs cannot be expressed as a single Jack polynomial, they still show a fine structure where the mathematical properties of the Jack polynomials play a major role.

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1. Introduction

Since the success of the Laughlin states [1], the use of trial wavefunctions in the fractional quantum Hall (FQH) effect has provided deep insights into these systems, especially non-Abelian

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ones [2,3]. Over the past few years there has been a renewed interest in non-Abelian states mostly because of their connection with topological quantum computing [4]. Model wavefunctions describing non-Abelian states can be constructed by using the conformal blocks of conformal field theories (CFTs). Much of the theory underlying the non-Abelian states is based on the monodromy properties of the conformal blocks.

The Read–Rezayi (RR) states [2,3], some of which are thought to be experimentally relevant, are a paradigm for non-Abelian states. These states are given by the conformal blocks of a particular family of CFTs, the so-called $\mathbb{Z}_k$ Fateev–Zamolodchikov (FZ) parafermionic field theories (defined below) [5]. These are CFTs with an extended cyclic $\mathbb{Z}_k$ symmetry to which corresponds a set of conserved current with a particular operator product expansion (OPE). The current OPEs define the so-called parafermionic algebras and the wavefunctions constructed by using the corresponding current correlation functions inherit specific clustering properties. In particular one can show that, apart from some gauge factor, the (bosonic) $\mathbb{Z}_k$ RR ground states are symmetric polynomials which vanish when at least $k+1$ particles come to the same point. The k-clustering properties make the $\mathbb{Z}_k$ RR states to be the unique densest zero-energy ground states of a $k+1$ body interaction Hamiltonians describing the energy cost to have $k+1$ particle occupying the same position [3,6]. In the case of the 2-body ($k=1$) interaction, the zero-energy ground state is the Laughlin state for bosons at filling fraction $\nu = 1/2$. The model Hamiltonian of this kind are believed to capture the physics of two-dimensional systems in very high magnetic fields where the effective Hamiltonian is reduced to the interaction between particles in acting in the lowest Landau level.

Because of the importance of the RR wavefunctions, an intense research activity has been focused on the generalizations of these functions and therefore of the FZ parafermionic theories. In addition to the k-clustering properties, symmetric polynomial are also characterized by the power r with which the polynomials vanish when the $k+1$st particle arrives. In terms of CFT, as we will see, the value of r determines the conformal dimension of the currents generating the $\mathbb{Z}_k$ symmetry. The RR states have $r=2$. As it was observed in [7], the $\mathbb{Z}_k$ Read–Rezayi wavefunctions, as well as other previously proposed non-Abelian wavefunctions [8], can be written in terms of a single Jack polynomial (Jacks, defined below) with negative parameter $\alpha = -(k+1)/(r-1)$. This has naturally suggested the possibility of describing quantum Hall wavefunctions in terms of Jacks. This approach has been the subject of a series of recent works [9–12] where the connection between Jacks and FQHE models has been studied in detail.

Interestingly, it was conjectured that the Jacks are directly related to correlators of certain CFTs based on the Lie algebra A_{k-1}, the so-called $WA_{k-1}(k+1, k+r)$ theories [9,13,14].

A crucial point is that for $r>2$, the $WA_{k-1}(k+1, k+r)$ theories are non-unitary as it is manifest from the negative value of their central charge c. However, there are solid arguments [15,16] that the wavefunctions constructed using non-unitary CFT cannot describe topological gapped quantum phases. In this respect, a recent work [17] has proposed that unitary Abelian theories may be built from non-unitary ones.

The initial observation that motivates the present work is that there is a family of CFTs, the parafermionic theories $\mathbb{Z}_k^{(r)}$ defined below, which include the $WA_{k-1}(k+1, k+r)$ theories as a special case. In particular, there exist $\mathbb{Z}_k^{(r)}$ theories based on a current algebra which is associative for each value of the central charge c, the $\mathbb{Z}_k^{(r)}(c)$ algebras. Analogously to the unitary sequence of minimal models based on the Virasoro algebra, the $\mathbb{Z}_k^{(r)}(c)$ algebras admit unitary representations for some discrete series of c values. We show that the correlators of $\mathbb{Z}_k^{(r)}(c)$ provide trial wavefunctions which satisfy for arbitrary c the same clustering properties as the