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Transition form factors of the pion in light-cone QCD sum rules with next-to-next-to-leading order contributions

S.V. Mikhailov [a], N.G. Stefanis [a,b,*]

[a] *Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Moscow Region, Russia*
[b] *Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany*

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Abstract

The transition pion–photon form factor is studied within the framework of light-cone QCD sum rules. The spectral density for the next-to-leading order corrections is calculated for any Gegenbauer harmonic. At the level of the next-to-next-to-leading order (NNLO) radiative corrections, only that part of the hard-scattering amplitude is included that is proportional to the β-function, taking into account the leading zeroth-order harmonic. The relative size of the NNLO contribution in the prediction for the form factor $F^{\gamma^*\gamma\pi}(Q^2)$ has been analyzed, making use of the BLM scale-setting procedure. In addition, predictions for the form factor $F^{\gamma^*\rho\pi}$ are obtained that turn out to be sensitive to the endpoint behavior of the pion distribution amplitude, thus providing in connection with experimental data an additional adjudicator for the pion distribution amplitude. In a note added, we comment on the preliminary high-Q^2 BaBar data on $F^{\gamma^*\gamma\pi}$ arguing that the significant growth of the form factor between 10 and 40 GeV^2 cannot be explained in terms of higher-order perturbative corrections at the NNLO.

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* Corresponding author at: Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany.
E-mail addresses: mikhs@phys.jinr.ru (S.V. Mikhailov), stefanis@tp2.ruhr-uni-bochum.de (N.G. Stefanis).

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1. Introduction

Although higher-order calculations in QCD perturbation theory have already a long history, little is known about exclusive processes at the next-to-leading order (NLO) level [1–6], and beyond [7,8], because these are quite complex in detail. In view of more and more high-precision experimental data for a variety of hadronic processes becoming gradually available, the importance of such higher-order calculations exceeds the pure theoretical interest and acquires phenomenological relevance. In particular, processes with two photons in the initial state, one far off-shell and the other quasi real,

$$\gamma^* + \gamma \to \pi^0,$$

provide a useful tool to access (after their fusion) the partonic structure of the produced hadronic states, e.g., pseudoscalar mesons.

Experimentally, the photon-to-pion transition form factor within this class of two-photon processes has been measured by the CLEO Collaboration [9] with high precision and extending the range of Q^2 up to 9 GeV^2, as compared to the previous low-momentum CELLO data [10]. Theoretically, this high precision allows one to test models and fundamental quantities, like the pion distribution amplitude (DA), the applicability of QCD factorization, etc.—see [5,6,11–27] and references cited therein. Moreover, one can determine [26] a compatibility region between the CLEO data and constraints derived from lattice simulations on the second moment of the pion DA [28,29]. This information can then be used to extract a range of values of the fourth moment of the pion DA that would simultaneously fulfil both constraints (CLEO and lattice). This prediction [26] can provide a guide for the determination of this moment on the lattice, a task that has not been accomplished yet.

For two highly virtual photons, perturbative QCD works well because factorization at some factorization scale μ_F^2 applies, so that the process can be cast into the form of a convolution

$$F^{\gamma^*\gamma^*\pi}\big(Q^2, q^2\big) = C\big(Q^2, q^2, \mu_\mathrm{F}^2, x\big) \otimes \varphi_\pi\big(x, \mu_\mathrm{F}^2\big) + \mathcal{O}\big(Q^{-4}\big), \tag{1.1}$$

which contains a hard part C, calculable within perturbation theory, and a wave-function part φ_π that is the (leading) twist-two pion distribution amplitude [30] and has to be modeled within some nonperturbative framework (or be extracted from experiment). Here, the omitted twist-four contribution represents subleading terms in the operator product expansion (OPE), which are suppressed by inverse powers of the photon virtualities.

To be more precise, consider the hard process of two colliding photons producing a single pion, $\gamma^*(q_1)\gamma^*(q_2) \to \pi^0(p)$, which is defined by the following matrix element [14]

$$\int d^4x\, e^{-iq_1\cdot z}\big\langle\pi^0(p)\big|T\big\{j_\mu(z)j_\nu(0)\big\}\big|0\big\rangle = i\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta F^{\gamma^*\gamma^*\pi}\big(Q^2, q^2\big), \tag{1.2}$$

where $Q^2 = -q_1^2$, $q^2 = -q_2^2$ denote the virtualities of the photons, $\pi^0(p)$ is the pion state with the momentum $p = q_1 + q_2$, and $j_\mu = (\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d)$ is the quark electromagnetic current. This process is illustrated graphically in the left panel of Fig. 1 and has been examined theoretically, for instance, in [1–3,17,31].

If both virtualities, Q^2 and q^2, are sufficiently large, the T-product of the currents can be expanded near the light cone ($z^2 = 0$) by virtue of the OPE to obtain the well-known leading-