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Mean-field gauge interactions in five dimensions I. The torus

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Abstract

We consider the lattice regularization of a five-dimensional $SU(2)$ gauge theory with periodic boundary conditions. We determine a consistent mean-field background and perform computations of various observables originating from fluctuations around this background. Our aim is to extract the properties of the system in regimes of its phase diagram where it seems to be in a dimensionally reduced state. Within the mean-field theory we establish the existence of a second order phase transition at finite value of the gauge coupling for anisotropy parameter less than one, where there is evidence for dimensional reduction.

1. Introduction

It is possible that our world has more than four space–time dimensions. There are different ways that extra dimensions could leave a trace depending, among other things, on which of the fundamental forces we choose to look at. We will concentrate here on the gauge interactions. Leaving to the side for the moment the fermionic sector we present here our investigation of the dynamics of extra-dimensional pure gauge theories with focus on dimensional reduction. More specifically, we would like to propose a scheme where this phenomenon can be analyzed analytically, far from the perturbative regime.

The phase diagram of a five-dimensional $SU(N)$ gauge theory in infinite four-dimensional spatial volume is parametrized by the two dimensionless couplings $\beta = 2N/(g_5^2 \Lambda)$ and $N_5 = 2\pi R\Lambda$, with g_5 the five-dimensional bare coupling, R the physical length parametrizing the size

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of the fifth dimension and Λ a cut-off.[1] The domain of weak coupling perturbation theory is the vicinity of the "trivial" point β, $N_5 \to \infty$ of the phase diagram which is approached as one removes the cutoff. From the formal point of view, up to now most analytical investigations of higher-dimensional theories have been carried out in this domain, which however comes with two caveats: as one approaches the fixed point, physics is governed by the triviality of the coupling and as one tries to enter in the interior of the phase diagram cut-off effects dominate. Far from the trivial point, in the small β regime, the system finds itself in a confined phase below a critical value β_c of order one. The regime $\beta_c < \beta \ll \infty$ where neither perturbation theory nor the strong coupling expansion is useful, up to now, is analytically basically unexplored.

The success of the Standard Model (SM) in explaining experimental data requires, after introducing a new ingredient, a natural way to sufficiently hide it. As far as higher-dimensional theories are concerned this would mean either that the extra dimensions are very small or that there is a four-dimensional slice in the higher-dimensional space where at least part of the physics is localized. From the phenomenological point of view the former situation has been also investigated by standard methods, mainly with a combination of Kaluza–Klein theory and weak coupling perturbation theory [1]. In this approach the size of the extra dimension(s) can be tuned to small values and the delicate issue is to make it small enough so that it does not lead to a contradiction with the data but large enough so that it can have a sizable effect. In the localization scheme, the extra dimension can be, in principle, as large as any other dimension (or even larger) but a dynamical mechanism is necessary to implement it. Unlike for gravity, for gauge fields a classical localization mechanism is not known and perturbation theory does not seem to help in this respect. Consequently, in most model building approaches when a certain field needs to be localized (for some phenomenological reason) one just assumes that it is. Nevertheless, a few non-perturbative quantum mechanisms of gauge field localization were invented in the past [2,3]. In these, lattice Monte Carlo simulations have been proven a crucial tool. Despite all efforts, from the five-dimensional point of view a quantum field theoretical approach to the localization of non-Abelian gauge fields has been elusive.

This state of affairs calls for a complementary tool to the perturbative computations and the numerical simulations, one which would enable us to probe the system analytically up to its phase transition. The method known to work in the domain of β of order one is the mean-field approximation [4,5]. This approach has been used before to locate the critical value of β_c for various gauge theories on toroidal geometries (see [4] and references therein) and in [6] it was used to locate β_c for an orbifold geometry. In the 80s there was a considerable effort to define an expansion in the fluctuations around the saddle-point (or zeroth order) approximation [7]. The free energy was indeed computed to a high order in order to locate any phase transitions very precisely and hopefully predict their order. However, extensive observables other than the free energy were never computed systematically especially for theories of large dimensionality.

In fact, it is believed that the expansion around the mean-field background becomes a better and better approximation to the non-perturbative behavior of the system as the number of space–time dimensions increases so it seems that it is just tailored for our purposes. In addition, we need a regulator that maintains a finite cutoff while preserving gauge invariance. The computational scheme will be therefore a lattice regularization with lattice spacing $a = 1/\Lambda$ in the mean-field approximation to zeroth order. At higher orders we will perturb away from the mean-field background by corrections that indeed go as one over the number of space–time dimensions. For

[1] Later we will be interested in anisotropic spaces where a third parameter, γ, will appear.