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Masses, fermions and generalized D-dimensional unitarity

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Abstract

We extend the generalized D-dimensional unitarity method for numerical evaluation of one-loop amplitudes by incorporating massive particles. The issues related to extending the spinor algebra to higher dimensions, treatment of external self-energy diagrams and mass renormalization are discussed within the context of the D-dimensional unitarity method. To validate our approach, we calculate in QCD the one-loop scattering amplitudes of a massive quark pair with up to three additional gluons for arbitrary spin states of the external quarks and gluons.

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1. Introduction

Good understanding of background and signal processes will be necessary to interpret data from the Large Hadron Collider (LHC) and observe physics beyond the Standard Model. In particular, large multiplicity final states are of interest [1]. Reliable predictions for such processes require computations of next-to-leading order (NLO) QCD corrections. Traditional methods for NLO calculations have difficulties in dealing with processes of such complexity; as a result, many new approaches to one-loop computations have been suggested in recent years [1].

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Among those approaches, generalized unitarity stands out [2–24]. The key feature of this method is that it allows calculation of one-loop scattering amplitudes directly from tree amplitudes leading to a computational algorithm of polynomial complexity [25]. The efficiency of generalized unitarity for NLO calculations for processes with high multiplicity final states has been explicitly demonstrated in Refs. [26,27].

Until recently, generalized unitarity was mostly used to compute the cut-constructible parts [28] of scattering amplitudes, while calculations of the rational parts proved to be challenging. In Refs. [10,11] the four-dimensional boot-strap method was developed to evaluate the rational part. Another approach developed to generate the rational part uses generalized D-dimensional unitarity [15,16].

In a recent paper [24], we extended the method of Refs. [23,29] in such a way that *both* cut-constructible and rational parts are obtained within a single formalism using integer-dimensional on-shell cuts. This method leads to a computational algorithm of polynomial complexity, as shown in Ref. [27].

Up to now, generalized unitarity has been mainly studied in the context of multi-gluon scattering amplitudes which simplifies the problem significantly. In the general case, one has to deal with two additional issues – different types of particles that participate in the scattering process and the fact that massive particles can be involved. It is necessary to address these issues before generalized unitarity becomes a practical tool for NLO calculations of phenomenological interest. The goal of this paper is to do exactly that and extend the applicability of generalized D-dimensional unitarity by considering one-loop amplitudes involving gluons and massive quarks. The computational method developed in Ref. [24] can handle both extensions easily.

Dealing with particles of different flavors requires more sophisticated bookkeeping, but is otherwise straightforward. However, the presence of massive particles introduces new conceptual issues. An obvious consequence of having virtual particles with non-zero masses contributing to one-loop scattering amplitudes is that in addition to quadruple, triple and double cuts, we also have to deal with single-particle cuts. Such an extension is straightforward; the necessary details have already been given in Ref. [23]. A more interesting consequence of massive particles present in the scattering process is that generalized unitarity applied to certain double- and single-particle cuts becomes more subtle. This is closely related to external wave function renormalization constants which originate from Feynman diagrams with self-energy insertions on external lines.[1] We will show that this complication can be circumvented without encumbering the formalism.

To validate the method, we focus on the calculation of one-loop amplitudes with a massive quark–anti-quark pair and up to three gluons. These one-loop amplitudes have been calculated using more traditional methods. The one-loop corrections to $t\bar{t}+2$ and $t\bar{t}+3$ partons scattering have been first calculated in Ref. [30,31] and Ref. [32], respectively.

The outline of the paper is as follows. In Section 2 we discuss the modification of the D-dimensional generalized unitarity method required to include massive fermions. Section 3 describes the subtleties that arise when massive particles are involved in the one-loop scattering amplitude. In Section 4 we present numerical results for the one-loop amplitudes $0 \mapsto t\bar{t}+2$ gluons and $0 \mapsto t\bar{t}+3$ gluons. The conclusions and outlook are given in Section 5.

[1] Similar problems appear due to diagrams that can be interpreted as one-loop expectation values of quantum fields.