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On-shell renormalization of the mixing matrices in Majorana neutrino theories

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Abstract

We generalize a recently proposed on-shell approach to renormalize the Cabibbo–Kobayashi–Maskawa quark-mixing matrix to the case of an extended leptonic sector that includes Dirac and Majorana neutrinos in the framework of the seesaw mechanism. An important property of this formulation is the gauge independence of both the renormalized and bare lepton mixing matrices. Also, the texture zero in the neutrino mass matrix is preserved.

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1. Introduction

Renormalizability endows the Standard Model (SM) with enhanced predictive power due to the fact that ultraviolet (UV) divergences from quantum effects can be eliminated by a redefinition of a finite number of independent parameters, such as masses and coupling constants. Furthermore, it has been known for a long time that, in the most frequently employed formulations in which the complete bare mass matrices of quarks are diagonalized, the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix [1] must be also renormalized [2]. In fact, this problem has been the object of several interesting studies over the last two decades [3,4].

A matter of considerable interest is the generalization of these considerations to minimal renormalizable extensions of the SM. In particular, in Refs. [5,6] the mixing-matrix renormal-

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ization was extended to theories that include isosinglet neutrinos and admit the presence of lepton-number-violating Majorana masses. A minimal realization of such a theory is the SM with right-handed Dirac and Majorana neutrinos [5,7], an appealing scenario that may explain the smallness of the observed neutrino masses and may lead to neutrino-less double beta decays. Furthermore, this minimal extension may give rise to a number of observable phenomena, such as lepton-flavor and/or lepton-number violation in μ, τ [8] and Z-boson decays [9], or to possible lepton-number-violating signals at high-energy colliders [10].

The aim of this paper is to generalize the on-shell renormalization of the CKM matrix recently proposed in Ref. [4] to extensions of the SM in which the lepton sector contains Majorana neutrinos. An important property is that this formulation complies with UV finiteness and gauge independence,[1] and also preserves the basic structure of the theory. In particular, the texture zero ($m_L'^0 = 0$) in the neutrino mass matrix is preserved by renormalization.

This paper is organized as follows. After briefly reviewing in Section 2 the basic formalism of the seesaw mechanism in the minimal extension of the SM neutral-lepton sector, we evaluate in Section 3 the one-loop self-energy insertions (see Figs. 1 and 3) in an external charged-lepton or Majorana-neutrino leg, perform the separation into wave-function renormalization (wfr) and self-mass (sm) amplitudes, and show explicitly the cancellation of gauge dependences in the latter. As in the quark case [4], the mass counterterm matrix, to be discussed in Section 4, is chosen to cancel, as much as possible, the sm contributions. In Section 5, we discuss the diagonalization of the complete mass matrix and show explicitly how this procedure generates mixing counterterm matrices in a manner that preserves the basic structure of the theory, as well as gauge independence and UV finiteness. Finally, our conclusions are summarized in Section 6.

2. Neutrino seesaw mechanism

We consider a minimal, renormalizable extension of the SM, based on the $SU(2)_I \otimes U(1)_Y$ gauge group, that can naturally accommodate heavy Majorana neutrinos. We allow for an arbitrary number N_G of fermion generations. Similarly to the SM, each lepton family contains one weak-isospin (I) doublet $(\nu_{L,i}'^0, l_{L,i}'^0)$ of left-handed states with weak hypercharge $Y = -1$ and one right-handed charged-lepton state $l_{R,i}'^0$ with $I = 0$ and $Y = -2$ ($i = 1, 2, \ldots, N_G$). In addition, there is a total of N_R right-handed neutrinos $\nu_{R,i}'^0$ with $I = Y = 0$ ($i = 1, 2, \ldots, N_R$). The superscript 0 denotes bare quantities, while the primes are to remind us that we are dealing with weak-interaction eigenstates.

The bare Lagrangian density contains the neutrino mass terms

$$\mathcal{L}'^{0,\nu} = -\frac{1}{2}\left(\bar{\nu}_L'^0, \bar{\nu}_R'^{0C}\right) m'^{0,\nu} \begin{pmatrix} \nu_L'^{0C} \\ \nu_R'^0 \end{pmatrix} + \text{h.c.}, \tag{1}$$

where $\nu_L'^0 = (\nu_{L,1}'^0, \ldots, \nu_{L,N_G}'^0)^T$, $\nu_R'^0 = (\nu_{R,1}'^0, \ldots, \nu_{R,N_R}'^0)^T$, the superscript C denotes charge conjugation, T means transpose, and $m'^{0,\nu}$ is a complex, symmetric matrix of the form

$$m'^{0,\nu} = \begin{pmatrix} m_L'^0 & m_D'^0 \\ m_D'^{0T} & m_M'^0 \end{pmatrix}. \tag{2}$$

[1] Throughout this paper, the term *gauge independence* is used as an abbreviation for *gauge parameter independence*.