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The GL(1|1)-symplectic fermion correspondence

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Abstract

In this note we prove a correspondence between the Wess–Zumino–Novikov–Witten model of the Lie supergroup GL(1|1) and a free model consisting of two scalars and a pair of symplectic fermions. This model was discussed earlier by LeClair. Vertex operators for the symplectic fermions include twist fields, and correlation functions of GL(1|1) agree with the known results for the scalars and symplectic fermions. We perform a detailed study of boundary states for symplectic fermions and apply them to branes in GL(1|1). This allows us to compute new amplitudes of strings stretching between branes of different types and confirming Cardy's condition.

1. Introduction

Conformal field theories with supersymmetric target space has become an important area of current research. They are essential in a variety of significant problems both in string theory and in disordered systems.

Understanding sigma models on supersymmetric spaces deep in the strongly coupled regime is of primary importance. In many models one believes that there exists a dual description which is better accessible in such a regime. The most prominent example is certainly the celebrated AdS/CFT correspondence [1,2], but there are, of course, other interesting dualities involving sigma models on supersymmetric spaces. For example, recently a strong-weak duality between the $\mathrm{OSp}(2N+2|2N)$ Gross–Neveu model and the principal chiral model on the supersphere $S^{2N|2N+1}$ was conjectured [3,4].

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There are various ways to find and to test such correspondences. Many supersymmetric spaces possess a family of conformally invariant field theories, and points in the moduli space that are exactly solvable e.g. the Wess–Zumino–Novikov–Witten point on supergroup manifolds or the infinite radius limit of the principal chiral model on the supersphere. In these cases, one way to test a duality is to compute certain quantities, e.g. some boundary spectra, at this solvable point and perform the perturbation to other points in the moduli space exactly [5]. This method has successfully been applied to the supersphere/Gross–Neveu correspondence [6]. The question remains how to actually prove such a correspondence. The case $N = 0$ in the Gross–Neveu-supersphere duality is the well-known correspondence between the O(2) Gross–Neveu model, that is the massless Thirring model, and a free boson on the circle, i.e. bosonization [7]. Unfortunately, the proof does not generalize straightforwardly, but still we believe that bosonization techniques will turn out to be crucial in understanding the correspondence.

If there is a simpler model at hand, it is a good idea to study it in detail to gain insight and to establish techniques for the more complicated models. This leads us to the GL(1|1)-symplectic fermion correspondence. The GL(1|1) WZNW model is probably the best understood CFT with supersymmetric target space that is not free. There exists another CFT with GL(1|1) current symmetry [8], which was used to study the spin quantum Hall transition. This CFT is constructed from the OSp(2|2) Gross–Neveu model at the free point via bosonization and it automatically has GL(1|1) symmetry since the OSp(2|2) Gross–Neveu model is constructed from a spin one half vector transforming in the adjoint representation of GL(1|1). This model consists of two free scalars and a set of symplectic fermions. The symplectic fermions were first analyzed in detail in [9,10]. The first part of this note is devoted to showing the correspondence between these models. The technique we use is based on bosonization, but in addition we use the affine currents as a guideline which we hope is also useful for other Gross–Neveu-like models.

The correspondence, we find, is remarkable in its own right since the GL(1|1) WZNW model is an interacting theory, while the corresponding model is free, and the bosons are completely decoupled from the fermions. The non-triviality is hidden in the vertex operators, i.e. the GL(1|1) vertex operators in the free description contain twist fields of the fermions and it turns out that the computation of bulk correlation functions in both descriptions is of a similar complexity. Still our method provides a new approach to WZNW models on Lie supergroups. So far, the models have been investigated either algebraically [11] or in terms of fermionic ghost systems [12–15]. Hopefully, there exist generalizations of our approach to other Lie supergroups leading to a better understanding of them.

In the second part of this note, we apply the correspondence to branes in GL(1|1). For the understanding of Cardy boundary states, the free description is better adapted than the original one. GL(1|1) possesses two classes of branes. One of them, the so-called untwisted branes which geometrically describe superconjugacy classes in the supergroup manifold, have been studied in detail in [16]. It was found that amplitudes of boundary states satisfy Cardy conditions [17] and that they agree with fusion, as expected from experience with rational CFT [18] and also logarithmic CFT [19]. The second class of branes contains just one volume-filling brane. This brane has been investigated in [15], i.e. its correlation functions have been computed, but also the boundary state has been constructed and tested. For a complete description of boundary states one still needs to understand amplitudes of strings stretching between a twisted and an untwisted brane. In our new description this can be done straightforwardly. The final result is that Cardy conditions are still satisfied, and essentially all known results of branes on Lie groups carry over to the Lie supergroup GL(1|1).