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Towards holographic renormalization of fake supergravity

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Abstract

A step is made towards generalizing the method of holographic renormalization to backgrounds which are not asymptotically AdS, corresponding to a dual gauge theory which has logarithmically running couplings even in the ultraviolet. A prime example is the background of Klebanov–Strassler (KS). In particular, a recipe is given how to calculate renormalized two-point functions for the operators dual to the bulk scalars. The recipe makes use of gauge-invariant variables for the fluctuations around the background and works for any bulk theory of the fake supergravity type. It elegantly incorporates the renormalization scheme dependence of local terms in the correlators. Before applying the method to the KS theory, it is verified that known results in asymptotically AdS backgrounds are reproduced. Finally, some comments on the calculation of renormalized vacuum expectation values are made.

1. Introduction

The string/gravity correspondence allows to calculate correlators in certain strongly coupled gauge theories via solving the equations of motion of supergravity (SUGRA). Similar to calculations in field theory, this requires regularization and renormalization. In the case of asymptotically AdS-spaces, the method of holographic renormalization (HR) has been developed

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systematically in [1–9]. However, these methods do not cover cases in which the field theory has a logarithmically running coupling even in the ultraviolet (UV).[1] Holographically, this translates to a bulk metric which is not asymptotically AdS (aAdS), but has logarithmic warping (in a suitably defined radial variable). The prime example of such a background is the Klebanov–Strassler (KS) solution [11], which is well approximated in the UV by the Klebanov–Tseytlin (KT) solution [12]. Calculating correlation functions in these cases is much more involved, also because the procedure of HR has not been worked out yet in a systematic way similar to the aAdS case.

As a result, only a few attempts to calculate correlators using the KT background have been made, cf. [13–16], and only in [15,16] the program of HR, as reviewed in [7], was applied. Furthermore, calculations of mass spectra in the KS background [13,17–21] have been done using a pragmatic approach assuming that a consistent method of HR in aAdS backgrounds exists.

In this note, we would like to readdress the question of how to calculate renormalized correlators holographically from backgrounds which are not aAdS, given that this is a feature one would expect for the dual description of any gauge theory with a running coupling in the UV, like QCD. Our approach is different in spirit from [15,16]. We consider a general bulk theory of gravity coupled to an arbitrary number of scalars, whose potential can be expressed via a "superpotential". Such theories are known as "fake SUGRA" theories [22][2] and allow for BPS domain wall background solutions, which are the holographic duals of renormalization group flows. The fake SUGRA systems include the case of KS (and also KT), when viewed as a consistent truncation of type-IIB SUGRA [25,26]. For such a general theory, it would be a daunting task to find the complete counterterms. Thus, we take a step back and content ourselves with giving a recipe how to calculate renormalized two- (and to some extent one-) point functions of the operators dual to the scalars of the theory. Furthermore, we only consider field theories living on a flat space–time, which allows us to ignore all counterterms involving the space–time curvature. In a sense, our approach is inspired by [8,9], where the philosophy was put forward to concentrate on the part of the counterterm action which is really necessary to calculate n-point functions for a given n, i.e., the terms of nth order in the fluctuations. In this spirit, we consider the case $n = 2$. The counterterms we propose involve the fluctuations in a covariant way, but, otherwise, do depend on the background. It might be possible to derive them from a fully covariant expression, but we have not attempted to do so.

The starting point of the holographic calculation of correlation functions in AdS/CFT is the correspondence formula [27]

$$\mathrm{e}^{-S_{\text{on-sh}}[\mathfrak{s}]} = \int \mathcal{D}\Phi\, \mathrm{e}^{-S_{\text{QFT}}[\Phi]+\int \mathcal{O}_i \mathfrak{s}_i \,\mathrm{d}^d x}, \tag{1.1}$$

where $S_{\text{on-sh}}[\mathfrak{s}]$ denotes the *renormalized* bulk on-shell action evaluated as a functional of suitably defined boundary values $\mathfrak{s}_i$ of the various bulk fields, which are identified with the *sources* coupling to certain QFT operators $\mathcal{O}_i$ (we will make this more precise below). Hence, the bulk quantity $S_{\text{on-sh}}[\mathfrak{s}]$ is identified with the generating functional of the connected correlation functions of various QFT operators. In particular, the *exact* one-point functions of the QFT operators

[1] Recently, progress was made on the holographic renormalization in bulk backgrounds which are conformal to $AdS_{p+2} \times S^{8-p}$ with a non-vanishing dilaton [10]. These cases imply couplings that run with a power law in the UV.

[2] Here "fake" does not mean that the theory is necessarily non-supersymmetric, just that the formalism is applicable more generally. The relation between supergravity and fake supergravity was analyzed in [23,24].