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Dominant NNLO corrections to four-fermion production near the W-pair production threshold

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Abstract

We calculate the parametrically dominant next-to-next-to-leading order corrections to four-fermion production $e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d} + X$ at centre-of-mass energies near the W-pair production threshold employing the method of unstable-particle effective theory. In total the correction is small, leading to a shift of 3 MeV in the W-mass measurement. We also discuss the implementation of realistic cuts and provide a result for the interference of single-Coulomb and soft radiative corrections that can easily be extended to include an arbitrary number of Coulomb photons.

1. Introduction

W-pair production at $e^- e^+$ colliders is a key process for measuring the W-boson mass and testing the non-abelian structure of the Standard Model (SM), because of its sensitivity to triple non-abelian gauge couplings. The total cross section has been measured at LEP2 in a kinematical region spanning from the W-pair production threshold to a centre-of-mass energy of 207 GeV [1] with an accuracy of $\sim 1\%$ at the highest energies. The W mass has been determined with an error of ~ 40 MeV reconstructing the W bosons from their decay products; bounds on possible anomalous couplings are less stringent, and deviations from the SM predictions have been constrained at the per-cent level. A detailed analysis of the W-pair production process will be possible at the planned International Linear Collider, where the total cross section could be measured at the permille level [2]. The precision on the W-mass determination has been estimated to be ~ 10 MeV

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by a direct reconstruction of the W-decay products [3], and ~ 6 MeV by a dedicated threshold scan [4]. Moreover, the possibility to scan the high-energy region would allow to measure more precisely the size of the triple non-abelian gauge couplings.

The estimates on the W-mass determination rely on statistics and the performance of the future linear collider, and they assume that the cross section can be predicted by theory to sufficient accuracy in order to convert its measurement into one for the W mass. For this reason, radiative corrections to on-shell W-pair production have been thoroughly investigated in the past and are known at next-to-leading order (NLO) since the beginning of the 1980's [5]. However, the W bosons being unstable, a precise theoretical prediction has to be formulated for a final state of stable or sufficiently long-lived particles, represented by the fermion pairs produced by W decay, rather than for on-shell W bosons. NLO predictions for four-fermion production far from the W-pair threshold region are available since some time in the double-pole approximation [6–8] or with further simplifications [9,10]. Recently, a full NLO computation of four-fermion production has been performed in the complex-mass scheme [11,12] without any kinematical approximation; moreover, a compact analytic result around the threshold region has been obtained in [13] using effective field theory (EFT) methods [14–16].

In particular, the analysis performed in [13] led to the following conclusions: (1) a resummation of next-to-leading collinear logarithms from initial-state radiation is mandatory to reduce the error on the W mass from the threshold scan below ~ 30 MeV; (2) the NLO partonic cross section calculation in the EFT approach implies a residual error of $\sim$ 10–15 MeV. Although a large component of the uncertainty at point (2) can be removed using the full NLO four-fermion calculation [11,12], the computation of the dominant higher order corrections, whose contribution has been estimated to be roughly ~ 5 MeV in [13], is necessary to secure the 6 MeV accuracy goal [4].

In this paper we employ EFT techniques to calculate analytically the (parametrically) dominant next-to-next-to-leading order (NNLO) corrections to the inclusive four-fermion production process $e^-e^+ \to \mu^-\bar{\nu}_\mu u\bar{d} + X$, where X stands for an arbitrary flavour-singlet state, with a three-fold goal: to improve the EFT NLO calculation [13]; to derive a result which can be added on top of the full NLO prediction [11,12]; to reduce the uncertainty on the W-mass measurement below the required 5 MeV level by including a new set of higher order corrections.

The organisation of the paper is as follows. In Section 2 we outline the structure of our computation, review the essential features of the EFT method and identify the set of parametrically leading NNLO corrections. In Section 3 we describe in detail the calculation of the various contributions. In Section 4 we show the numerical impact of our result on the inclusive cross section and in Section 5 we discuss the effect of realistic cuts adopting those applied at LEP2 as a template. As a by-product we explain how invariant-mass cuts can be included in the EFT approach. Finally, Section 6 contains our conclusions and two appendices collect results related to the renormalisation of the Coulomb potential and the electromagnetic coupling, and the conversion from the fixed-width to the complex-mass scheme.

2. Outline of the computation

We consider the inclusive four-fermion production process

$$e^-(p_1)e^+(p_2) \to \mu^-\bar{\nu}_\mu u\bar{d} + X, \tag{1}$$

where X denotes an arbitrary flavour-singlet state (nothing, photons, gluons, ...), in the kinematical regime close to the W-pair production threshold, $s \equiv (p_1 + p_2)^2 \sim 4M_W^2$. Here the total