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# A new source for a brane cosmological constant from a modified gravity model in the bulk

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**Abstract**

We show that a four-dimensional equation of state for a cosmological constant term arises from a perfect fluid in the bulk in the context of a gravity model where the scalar curvature is non-minimally coupled to the perfect fluid Lagrangian density. The four-dimensional theory is fully determined from the induced equations on the brane, subject to the boundary conditions derived across the brane.


## 1. Introduction

Braneworld scenarios are an interesting development in what concerns gravity models and their cosmological implications [1]. Most often these scenarios assume, given observational constraints as well as theoretical assumptions, that the bulk space is empty except for a cosmological constant [2,3]. However, other scenarios have been proposed where the bulk is no longer empty but instead populated by matter fields [4–6]. More recently the implications of having vector and scalar fields in the bulk were studied in connection with Lorentz symmetry [7] and gauge symmetry breaking [8].

In this paper we consider the presence of a perfect fluid in the bulk in the context of a five-dimensional braneword model where the scalar curvature couples non-minimally to the La-

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grangian density of the perfect fluid. In $(3+1)$ dimensions this class of models with Lagrangian density of the form [9]

$$\mathcal{L} = \alpha f_1(R) + \big(1 + \lambda f_2(R)\big)\mathcal{L}_M, \tag{1}$$

where $f_1(R)$ and $f_2(R)$ are generic functions of the scalar curvature, was shown to exhibit interesting features which allow one to address problems such as the rotation curves of galaxies without the need of dark matter (see Ref. [9] and references therein) and the Pioneer anomaly (see Refs. [10,11] and references therein). The stability of these models has been examined in Ref. [12]. Other studies on their implications included their impact on stellar equilibrium and the analysis of their corresponding PPN parameters, which were studied in Refs. [13,14], respectively.

Recently there has also been interest in the conformal equivalence between $f(R)$ theories and Einstein gravity non-minimally coupled to a scalar field in the context of braneworlds [15,16]. The expected increasingly higher order of the discontinuity of the geometric quantities across the brane with the increasing power in $R$ of $f(R)$ is solved by enforcing continuity of the metric to correspondingly higher-order derivatives. Here, however, we will not impose further continuity conditions on the intervening fields, allowing for the discontinuity of the second derivative of the metric across the brane and orthogonal to its surface, despite also obtaining an increase in the power of $R$.

Crucial in the setting of our problem is a suitable implementation of the Israel matching conditions in the presence of bulk fields in order to extract the boundary conditions, both for gravity and the matter fields, which the induced equations of motion on the brane must satisfy. The method to be employed here was first introduced in Ref. [17] and further developed in Refs. [7,8]. For completeness and clarity, the more involved technical details of our method are presented in Appendix A. As we shall see, and rather remarkably, the projection of the bulk perfect fluid can induce on the brane a new cosmological constant term. This is one of the cases encompassed by the boundary conditions derived on a $\mathbb{Z}_2$-symmetric brane where conservation of energy is satisfied. This result suggests that a perfect fluid in the bulk space may have a bearing on the cosmological constant problem on the brane. This new source for a brane cosmological constant opens quite interesting perspectives for inflation at the early universe and for acceleration at the late time expansion of the universe.

This paper is organized as follows. In Section 2 we present our model and work out a suitable Lagrangian density for a perfect fluid. This development extends the approach of Hawking and Ellis [18] to the bulk space. In Section 3 we work out the matching conditions across the brane and derive the equations of motion therein induced. A derivation of the Gauss–Codacci relations is also presented in Appendix A for completion. Section 4 contains our results and Section 5 our conclusions.

## 2. A modified gravity model in the bulk

### 2.1. The Einstein equations

We consider the particular case of the action discussed in Ref. [9]. We set $f_1(R) = f_2(R) = R$ and introduce a cosmological constant as follows

$$S = \int d^5x\,\sqrt{-g}\big[M_{P(5)}^3 R + (1 + \lambda R)\mathcal{L}_M - 2\mathcal{L}_\Lambda\big]. \tag{2}$$