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Instantons and wormholes for the universal hypermultiplet

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Abstract

In this paper we present supersymmetric instanton and non-supersymmetric wormhole solutions for the universal hypermultiplet sector of $d = 4$ $N = 2$ supergravity theories. Instantons and wormholes are constructed as saddle points dominating transition amplitudes between states of definite axionic shift charge, using an approach due to Coleman and Lee. Our solutions are constructed in terms of the conserved Noether charges associated with the global $SU(2,1)$ symmetry of the universal hypermultiplet action. The conditions imposed by regularity on the charges are discussed.

1. Introduction

In string theory, instantons and wormholes can be constructed as extrema of the low energy supergravity Euclidean action [1–4].

It has been argued in the past that wormholes lead to bi-local terms in the low energy effective action and produce several interesting effects, such as renormalization of coupling constants and cosmological constant, quantum decoherence and creation of baby universes [2,3,5–9]. Wormholes in anti-de Sitter spaces have been investigated in [10–12], while some recent work [13] has pointed out how wormhole-induced effects determine a clash between the locality of an anti-de Sitter theory and the locality of the dual conformal field theory.

Instantons, on the other hand, lead to local non-perturbative contributions to the effective action. In supergravity theories, the BPS instanton solutions of the Euclidean equations of motion

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preserve half of the supersymmetries [15,16]. They can be viewed as an extremal limit of non-supersymmetric wormhole solutions, where the neck of the wormhole pinches off, leading to a local operator insertion.

The broken supersymmetries of the instanton solution generate fermionic zero modes. Transition amplitudes will vanish, unless the zero modes are soaked up by the insertion of appropriate operators in the path integral [16,17]. This leads to instanton-induced terms in the effective action, including four-fermion terms as well as corrections to the sigma model metric of the universal hypermultiplet. Instanton and wormhole solutions have been discussed for various theories and dimensions, in particular for the axion/dilaton $SL(2,R)/U(1)$ coset [14–16,18,19], the universal hypermultiplet in $N=2$, $d=4$ supergravity [20–26] and general hypermultiplets in $N=2$, $d=4$ theories [27–29].

In this paper we focus on instanton and wormhole solutions for the universal hypermultiplet sector of $N=2$ $d=4$ supergravities with vanishing cosmological constant. The universal hypermultiplet always appears as a subsector of the hypermultiplet sector of Calabi–Yau compactifications of type II string theories. In the case of type IIA on a so-called rigid Calabi–Yau (which has $h_{2,1}=0$), the universal hypermultiplet constitutes the complete hypermultiplet sector.

The main motivation of this paper is to apply the approach originally due to Coleman and Lee [4] to the universal hypermultiplet. In Section 2 we review the universal hypermultiplet and its symmetries. In Section 3 we generalize the approach from Coleman and Lee to the case of non-commuting shift charges. Instanton and wormhole amplitudes are constructed in the path integral framework as transition amplitudes between states of definite axionic shift charges. This allows us to obtain positive-definite Euclidean actions and clarifies the procedure of analytic continuation for axionic scalars. In Section 4 we construct the general instanton and wormhole solutions in terms of the conserved charges derived in Section 2. The extremal (BPS) limit is discussed in Section 5. The integrability of the supersymmetry variations is verified and the instanton action is calculated. In Section 6 we discuss the constraints on non-extremal wormhole solutions imposed by regularity. In Appendices A–C we present some detailed formulae for the NS–R charged solution, the relation of our solution to the one obtained by dualizing axionic scalars to tensor fields and the conserved charges after analytic continuation.

2. The universal hypermultiplet

The universal hypermultiplet action contains four scalar fields denoted with $\phi, \sigma, \zeta, \tilde{\zeta}$. The parameterization which is most useful for our purposes is given by:

$$S=\int d^4x\,\sqrt{g}\left\{\frac{1}{2}(\partial_\mu\phi)^2+\frac{1}{2}e^{-2\phi}(\partial_\mu\sigma+\tilde{\zeta}\,\partial_\mu\zeta)^2+\frac{1}{2}e^{-\phi}\left[(\partial_\mu\zeta)^2+(\partial_\mu\tilde{\zeta})^2\right]\right\}. \tag{2.1}$$

This action can be obtained by compactification of type IIA string theory on a rigid Calabi–Yau manifold with $h_{2,1}=0$. In all Calabi–Yau compactifications, it is possible to find a consistent truncation reducing the hypermultiplet action to one equivalent to the universal hypermultiplet.

The action (2.1) can also be written as a gauge-fixed sigma model action for the $\frac{SU(2,1)}{SU(2)\times U(1)}$ coset. The coset possesses a nonlinearly realized global $SU(2,1)$ symmetry. The eight infinitesimal generators $\Phi^i \to \Phi^i + \epsilon\delta_a\Phi^i$ are given[1] as follows. First, the shift symmetry of the NS–NS axion σ is generated by E, the simple shift of the R–R axion ζ is generated by E_q and the shift

[1] See Fig. 1 for the labelling of generators which is the same as in [30].