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The chaotic effects in a nonlinear QCD evolution equation

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Abstract

The corrections of gluon fusion to the DGLAP and BFKL equations are discussed in a united partonic framework. The resulting nonlinear evolution equations are the well-known GLR–MQ–ZRS equation and a new evolution equation. Using the available saturation models as input, we find that the new evolution equation has the chaos solution with positive Lyapunov exponents in the perturbative range. We predict a new kind of shadowing caused by chaos, which blocks the QCD evolution in a critical small x range. The blocking effect in the evolution equation may explain the Abelian gluon assumption and even influence our expectations to the projected Large Hadron Electron Collider (LHeC), Very Large Hadron Collider (VLHC) and the upgrade (CppC) in a circular e^+e^- collider (SppC).

1. Introduction

The QCD evolution equation is an important part in the study of high energy physics. The linear DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi) equation [1] and BFKL (Balitsky–Fadin–Kuraev–Lipatov) equation [2] are no longer reliable at ultra higher energy since the corrections of parton recombination. A series of nonlinear evolution equations, for example, the GLR–MQ–ZRS (Gribov–Levin–Ryskin, Mueller–Qiu, Zhu–Ruan–Shen) equation [3,4] and BK

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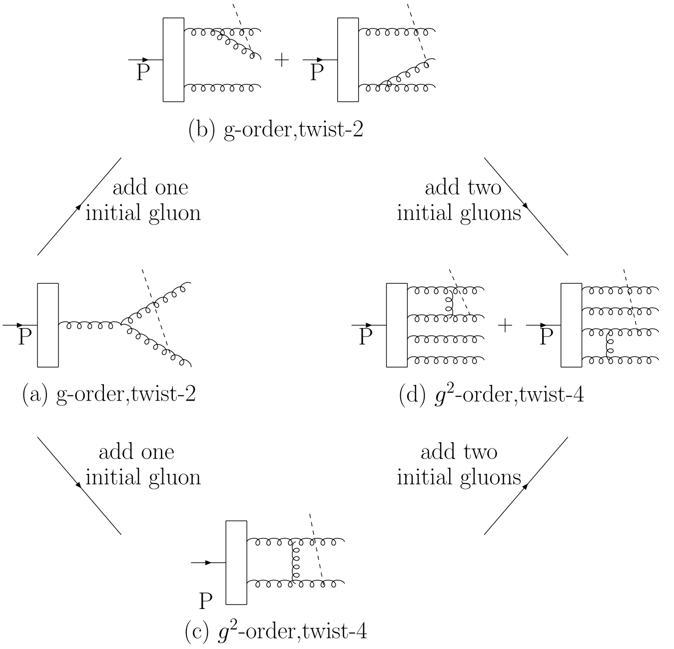

Fig. 1. The corrections of the initial gluons to a basic amplitude of the DGLAP equation (a) and they lead to (b) the BFKL equation, (c) the GLR–MQ–ZRS equation and (d) a new evolution equation, respectively. The dashed line is a virtual current which probing gluon. Note that the four evolution equations form a closed circuit, which implies a consistence among four evolution equations.

(Balitsky–Kovchegov) equation [5] were proposed, in which the corrections of parton recombination are considered.

As we know, the nonlinear iteration equations may have a characteristic solution-chaos, which has been observed in many natural phenomena [6]. A following question is: do the nonlinear QCD evolution equations have chaotic solution? Several years ago we have reported chaos in a new evolution equation [7], which describes the corrections of the gluon recombination to the BFKL equation at the leading logarithmic $LL(1/x)$ approximation. The purpose of this work is to detail this discovery after a long consideration.

We begin from the proposal of the new evolution equation. Fig. 1 is a schematic program, which shows that the correlations among initial gluons modify the evolution equations step by step. The elementary amplitude in Fig. 1a together with its conjugate amplitude constructs the DGLAP equation for gluon. The correlations among the initial partons are neglected in the DGLAP equation. This assumption is invalid in the higher density region of partons, where the parton wave functions begin to spatially overlap. Therefore, the corrections of the correlations among initial gluons to the elementary DGLAP amplitude at small x should be considered. To this end, we add the possible initial gluon lines in Fig. 1a step by step. The resulting three sets of amplitudes are listed in Fig. 1b–1d. It is interesting that these amplitudes produce the BFKL, GLR–MQ–ZRS equations and a new evolution equation.

We will present the derivations of the above mentioned four evolution equations in a same partonic framework. For this sake, we use the Bjorken frame, where the traditional parton distributions inside a fast moving target are defined in the factorization scheme. Note that the BFKL