Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B 911 (2016) 191–205

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

Sachdev–Ye–Kitaev model as Liouville quantum mechanics

Dmitry Bagrets [a,*], Alexander Altland [a], Alex Kamenev [b]

[a] *Institut für Theoretische Physik, Universität zu Köln, Zülpicher Straße 77, 50937 Köln, Germany*
[b] *W. I. Fine Theoretical Physics Institute and School of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455, USA*

Received 18 July 2016; accepted 2 August 2016

Available online 8 August 2016

Editor: Hubert Saleur

Abstract

We show that the proper inclusion of soft reparameterization modes in the Sachdev–Ye–Kitaev model of N randomly interacting Majorana fermions reduces its long-time behavior to that of Liouville quantum mechanics. As a result, all zero temperature correlation functions decay with the universal exponent $\propto \tau^{-3/2}$ for times larger than the inverse single particle level spacing $\tau \gg N \ln N$. In the particular case of the single particle Green function this behavior is manifestation of the zero-bias anomaly, or scaling in energy as $\epsilon^{1/2}$. We also present exact diagonalization study supporting our conclusions.

1. Introduction and the model

The Sachdev–Ye–Kitaev (SYK) model [1,2] is a system of N Majorana fermions, χ_j, $j = 1, \ldots, N$, subject to a four-fermion interaction,

$$\hat{H} = \frac{1}{4!} \sum_{ijkl}^{N} J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l \,, \tag{1}$$

* Corresponding author.
E-mail address: dmitry.bagrets@uni-koeln.de (D. Bagrets).

http://dx.doi.org/10.1016/j.nuclphysb.2016.08.002

where the coupling constants, J_{ijkl}, are independent Gaussian random variables distributed as

$$\langle J_{ijkl}\rangle = 0\,; \qquad \langle (J_{ijkl})^2\rangle = 3!J^2/N^3. \tag{2}$$

This seemingly innocent model has recently been recognized [2–4] as a possible shadow of a two-dimensional gravitational bulk and the ensuing perspective to explore a holographic correspondence of lowest possible dimension in concrete terms. The holographic principle as much as all other relevant features of the system are rooted in its exceptionally high level of symmetries. When formulated in the language of a $(0+1)$-dimensional 'field theory' the system shows approximate symmetry under reparameterization of time $\chi_j(\tau) \to \chi_j(f(\tau))$. In the limit of asymptotically slow time variations (compared to the characteristic interaction strengths J) that symmetry becomes exact. The infinite dimensional group of time reparameterizations is generated by a Virasoro algebra and for this reason the theory has been dubbed a 'nearly conformally invariant' (NCFT) theory, and the emerging holographic principle an $NAdS_2/NCFT_1$ correspondence.

Thanks to the presence of the large parameter N and the 'infinite rangedness' of the interaction the path integral over Majorana configurations can be processed by stationary phase methods. It turns out that the mean field, physically equivalent to a self-consistent approximation for an interaction self-energy, spontaneously breaks the conformal symmetry down to the three-dimensional group SL(2, R) of conformal transformations in one dimension. This sets the stage for the emergence of an infinite dimensional manifold of Goldstone modes described by reparameterization of time, $\tau \to f(\tau)$, modulo ordinary conformal invariance. The fluctuations of these modes are damped by explicitly symmetry breaking time derivatives which enter the theory in combinations $\sim \partial_\tau/J$ and play a role comparable to that of an external magnetic field in a ferromagnet. Due to the low dimensionality of the model all time-dependent correlation functions must be qualitatively affected by these fluctuations in the long time limit, where the breaking of symmetry becomes weak. In particular one should expect the mean-field amplitudes of all observables (such as e.g. Green functions) to be qualitatively changed at low frequencies, i.e. in the limit where the Mermin–Wagner theorem enforces a restoration of the full symmetry via proliferating Goldstone mode fluctuations.

Previous work [4] took a step towards an effective action $S[f(\tau)]$ for the reparameterization Goldstone modes and studied their effects to lowest (quadratic) order in perturbation theory. While this is sufficient to describe short time correlations, a different treatment is required in the infrared regime. In this paper we point out that the action $S[f]$ can be mapped onto the action of the *Liouville quantum mechanics*, i.e. quantum mechanics in an exponential potential [5–7]. This reformulation sets the stage for the treatment of the correlation functions in fluctuation dominated long time regime. We will show that at long times, $\tau J > N \ln N$, all correlation functions crossover to qualitatively different power laws. The ensuing long time asymptotics imply the vanishing of the mean-field two-point function at low frequencies. Where the four point function is concerned they represent operator correlations beyond those perturbatively (diagrammatically) described previously. Finally, the equivalence of the low frequency problem to Liouville quantum mechanics implies [8,9] that two-time correlation functions $\langle \mathcal{O}(\tau)\mathcal{O}(\tau')\rangle \sim |\tau - \tau'|^{-3/2}$ of arbitrary operators must decay with a universal 3/2-power law at large times. It stands to reason that this type of universality must reflect in the manifestations of a holographic correspondence in the infrared. However, the discussion of such correspondences is beyond the scope of the present paper and may become a subject of future work.