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# High-energy string scatterings of compactified open string

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**Abstract**

We calculate high-energy massive string scattering amplitudes of compactified open string. We derive infinite linear relations, or stringy symmetries, among soft high-energy string scattering amplitudes of different string states in the Gross kinematic regime (GR). In addition, we systematically analyze all hard power-law and soft exponential fall-off regimes of high-energy compactified open string scatterings by comparing the scatterings with their 26D noncompactified counterparts. In particular, we discover the existence of a power-law regime at fixed angle and an exponential fall-off regime at small angle for high-energy compactified open string scatterings. The linear relations break down as expected in all power-law regimes. The analysis can be extended to the high-energy scatterings of the compactified closed string, which corrects and extends the previous results in [J.C. Lee, Y. Yang, Linear relations and their breakdown in high energy massive string scatterings in compact spaces, Nucl. Phys. B 784 (2007) 22].


## 1. Introduction

There are three fundamental characteristics of high energy, fixed angle string scattering amplitudes [1–3], which are not shared by the field theory scatterings. These are the softer exponential fall-off behavior [1,4] (in contrast to the hard power-law behavior of field theory scatterings [5]), the infinite Regge-pole structure [6,7] of the form factor and the existence of infinite number of linear relations [6–17], or stringy symmetries, discovered recently among string scattering am-

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plitudes of different string states in the high-energy string scattering amplitudes. An important new ingredient to derive these linear relations is the zero-norm states (ZNS) [18–20] in the old covariant first-quantized (OCFQ) string spectrum. Other approaches related to this development can be found in [21–26].

It was believed [27,28] that the newly discovered linear relations are responsible for the softer exponential fall-off behavior of high-energy string scatterings. One way to justify this conjecture is to find more hard power-law high-energy string scatterings, which simultaneously show the breakdown of the above linear relations. However, it is well known that the genetic high-energy, fixed angle behavior of string scatterings is soft exponential fall-off rather than hard power-law. Recently, following an old suggestion of Mende [29], two of the present authors [28] calculated high-energy massive scattering amplitudes of closed bosonic string with some coordinates compactified on the torus. They obtained infinite linear relations among high-energy scattering amplitudes of different string states in the Gross kinematic regime (GR). This result is reminiscent of the existence of an infinite number of massive ZNS in the compactified closed string spectrum constructed in [30]. In addition, they discovered that, for some kinematic regime, these infinite linear relations break down and, simultaneously, the string amplitudes enhance to hard power-law behavior instead of the usual soft exponential fall-off behavior at high energies.

To further understand the relationship of the infinite linear relations and the softer exponential fall-off behavior of high-energy, fixed angle string scatterings, it is crucial to find more examples of high-energy string scatterings, which show the unusual power-law behavior and, simultaneously, give the breakdown of the infinite linear relations. In this paper, we calculate high-energy massive string scattering amplitudes of open bosonic string with some coordinates compactified on the torus. As in the case of compactified closed string, we obtain infinite linear relations among soft high-energy scattering amplitudes of different string states in the GR. This result is reminiscent of the existence of an infinite number of massive ZNS in the compactified open string spectrum constructed in [31]. More importantly, we analyze all possible hard power-law and soft exponential fall-off regimes of high-energy compactified open string scatterings by comparing the scatterings with their 26D noncompactified counterparts. In particular, we discover the existence of a power-law regime at fixed angle and an exponential fall-off regime at small angle for high-energy compactified open string scatterings. These new phenomena never happen in the 26D string scatterings. The linear relations break down as expected in all power-law regimes. The analysis can be extended to the high-energy scatterings of the compactified closed string, which corrects and extends the previous results in [28]. In particular, we correct the "Mende regime" discussed in [28], which is indeed exponential fall-off behaved rather than power-law claimed in [28]. As an example, we derive a hard power-law regime at fixed angle for high-energy compactified closed string scatterings. This paper is organized as follows. In Section 2 we calculated high-energy massive scattering amplitudes of compactified open string. In Section 3 we classify all kinematic regimes of the amplitudes and extend our results to the closed string case. A brief conclusion is given in Section 4.

## 2. High-energy scatterings

We consider 26D open bosonic string with one coordinate compactified on $S^1$ with radius $R$. As we will see later, it is straightforward to generalize our calculation to more compactified