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Variable cosmological constant model: The reconstruction equations and constraints from current observational data

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Abstract

In this paper we first give a brief review of the variable cosmological constant model and its scalar field description. We mainly discuss two types of variable cosmological constant models: a power law and H power law models. A method to obtain all of the equivalent scalar field potentials and the effective equation of state of the two models is presented. In addition, the dynamics of such scalar field potentials and effective equation of state are discussed in detail. The parameters of the two models are constrained by current 307 high-quality "Union" SN Ia data set, baryon acoustic oscillation (BAO) measurement from the Sloan Digital Sky Survey (SDSS), 9 observational $H(z)$ data derived from the Gemini Deep Deep Survey (GDDS) and the shift parameter of the cosmic microwave background (CMB) given by the three-year Wilkinson Microwave Anisotropy Probe (WMAP) observations. We also calculate and draw the picture of the Hubble parameter, the deceleration parameter and the matter density of the two models. Then, we show that the indices m and n in the two models have specific meaning in determining properties of the models. Moreover, the reasons that indices m and n may also influence the behavior of effective equation of state and scalar field potentials are presented.

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1. Introduction

In 1998, the discovery that the accelerated expansion of the Universe is driven by the dark energy (DE) from the type Ia supernovae (SN Ia) observations [1] greatly astonished the world. The Wilkinson Microwave Anisotropy Probe [2], combined with more accurate SN Ia data [3] indicates that the Universe is almost spatially flat and the dark energy accounts for about 70% of the total content of the Universe. However, we know little about the nature of dark energy except for its negative pressure. Therefore, a large number of works have been done in recent years to explain this mystery.

The variable cosmological constant (hereafter VCC) [4] is one of the phenomenological ways to explain the dark energy problem, because it is a straightforward modification of the cosmological constant Λ which enable itself to be compatible with observations. Looking back to the history, we can see that a lot of theorists have done numerous works to search for the theoretical foundation of the VCC models and also investigate the properties of the VCC models [5]. In [6], a model of $\Lambda \propto a^{-2}$ was proposed, requiring that the cosmic density ρ would equal to the Einstein–de Sitter critical density ρ_c, which leads to a closed Universe, without singularity, horizon, entropy and monopoly problems [6]. After that, it was also suggested a model $\Lambda \propto a^{-2}$ (Λ should be independent of $\hbar$) with different initial conditions by [7], which firstly pointed out that time-dependent Λ leads to the creation of matter or radiation. Besides, a lot of work were done to propose straightforward models relating Λ to the Hubble parameter $H(z)$: $\Lambda \propto H^2$ [8–11]. Furthermore, people also constructed a large number of phenomenal VCC models to describe the dynamics of the Universe and there is a list in Ref. [4] summarizing the proposed models [12]. There are also several papers concerning the observational constraints about the VCC models [13].

In addition to the VCC models, scalar fields such as "quintessence" [14], "phantom" [15] and "quintom" [16] have been introduced to effectively describe the dynamic dark energy, which are distinguished by the effective equation of state (hereafter EEoS): $w_{\mathrm{DE}} > -1$, $w_{\mathrm{DE}} < -1$ and w_{DE} across -1 respectively. These models are inspired by the fact that a decaying vacuum energy which has the very high energy density at early time should be sufficiently small at present to meet the current observation requirement, so they should evolve dynamically. In order to obtain the corresponding quintessence potentials, the reconstruction equations were derived and addressed the feasibility of the approach by Monte Carlo simulation [17]; it was also constructed the general scalar-field dark energy model [18] and developed a method to construct them directly from EEoS function $w_\phi(z)$ [19,20]. Moreover, some works have been done to reconstruct the scalar potential from the scalar-tensor theory and investigate the modified Newton theory [21].

As a major part of our work, we analyze the EEoS and reconstruct the potentials for two main types of VCC models—the a power law and the H power law models—from the point of view of dynamic scalar fields. This work is necessary for people who are interested in the coupled dark energy and dark matter [22], because such models may avoid a lot of realistic problems such as the coincidence problem. In addition, it is discussed how such phenomenological models can be explained as a classical scalar field decaying into a perfect fluid which might be interested by those who want to search for the gravitational theory other than the general relativity, because the Lagrangian in VCC should be different from the Einstein–Hilbert action in general relativity. Thus, this part should be essential for people to see the possible forms of VCC models and its corresponding scalar fields which are expected from string theory or supergravity.