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On q-deformed infinite-dimensional n-algebra

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Abstract

The q-deformation of the infinite-dimensional n-algebras is investigated. Based on the structure of the q-deformed Virasoro–Witt algebra, we derive a nontrivial q-deformed Virasoro–Witt n-algebra which is nothing but a sh-n-Lie algebra. Furthermore in terms of the pseud-differential operators, we construct the (co)sine n-algebra and the q-deformed $SDiff(T^2)$ n-algebra. We find that they are the sh-n-Lie algebras for the n even case. In terms of the magnetic translation operators, an explicit physical realization of the (co)sine n-algebra is given.

1. Introduction

Infinite dimensional Lie algebras have played a crucial role in physics. Much interest has been attributed to their q-deformed versions. It is well-known that the Virasoro algebra is an important infinite dimensional Lie algebra. Its q-deformation has been widely studied in the literature [1–10]. A q-deformation of the centerless Virasoro or Virasoro–Witt (V–W) algebra was

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first obtained by Curtright and Zachos [1]. Its central extension was later furnished by Aizawa and Sato [2]. Chaichian and Prešnajder [3] proposed a different version of the q-deformed Virasoro algebra by carrying out a Sugawara construction on a q-analogue of an infinite dimensional Heisenberg algebra. Shiraishi et al. [4] presented a q-Virasoro algebra $Vir_{q,t}$, where q and t are two complex parameters. They constructed a free boson realization of this q-Virasoro algebra and showed that singular vectors can be expressed by the Macdonald symmetric functions. It is similar to the case of the ordinary Virasoro algebra whose singular vectors are given by the Jack symmetric functions. It is well-known that there is a remarkable connection between the Virasoro algebra and the Korteweg–de Vries (KdV) equation [11,12]. For the q-deformed Virasoro algebra, Chaichian et al. [13] showed that it generates the symplectic structure which can be used for a description of the discretization of the KdV equation. Furthermore the quantum KdV equations associated with the algebraic symmetry have been investigated in Refs. [14,15]. The integrable one-dimensional quantum spin chains have attracted much interest from physical and mathematical points of view. One noted that the deformed Virasoro algebra plays an important role in the study of the XYZ model [16,17].

The W_N algebras are extensions of the Virasoro algebra which contain generators of all conformal spins from $s = 2$ up to $s = N$. The W_∞ algebra may arise in an appropriate large N limit of W_N. The q-deformations of W_N and W_∞ algebras have been well investigated [6,18–21]. Recently Taki [22] proposed the generalized Alday–Gaiotto–Tachikawa–Wyllard (AGT-W) correspondence between $5d$ uplift of $4d$ $N = 2$ $SU(N)$ asymptotically-free gauge theories and the q-deformed W_N algebra. It was found that the Nekrasov partition function of a $5d$ gauge theory is equal to the scalar product of the corresponding Whittaker vectors of the q-deformed W_N algebra. The contraction of the W_∞ algebra leads to the so-called w_∞ algebra which is equivalent to the algebra of smooth area-preserving diffeomorphisms of the cylinder $S^1 \times R^1$ [23]. It is worth to emphasize that the algebra of the area-preserving diffeomorphisms of the torus T^2, i.e., the so-called $SDiff(T^2)$ algebra [24,25] is also an important infinite-dimensional algebra. In terms of the Gauss derivatives on the quantum plane, Kinani et al. [26] presented the q-deformed $SDiff(T^2)$ algebra. It should be noted that the sine algebra arises as the unique Lie algebra deformation of $SDiff(T^2)$ in some suitable basis. There has been considerable interest in the (super) sine algebra [27–30].

The Nambu 3-algebra was introduced in [31,32] as a natural generalization of a Lie algebra for higher-order algebraic operations. Recently Bagger and Lambert [33,34], and Gustavsson [35] (BLG) found that 3-algebras play an important role in the world-volume description of the multiple M2-branes. Due to BLG theory, there has been considerable interest in the 3-algebra and its application. More recently there has been the progress in constructing the infinite-dimensional 3-algebras, such as V–W [36,37], Kac–Moody [38] and w_∞ 3-algebras [39,40]. It is well-known that the infinite-dimensional algebras have a deep intrinsic connection to the integrable systems. Recently the relation between the infinite-dimensional 3-algebras and the integrable systems has also been studied in the framework of Nambu mechanics [41,42].

Recently Curtright et al. [36], constructed a V–W algebra through the use of $su(1,1)$ enveloping algebra techniques. It is worthwhile to mention that this ternary algebra depends on a parameter z and is only a Nambu–Lie algebra when $z = \pm 2\mathbf{i}$. Ammar et al. [43] presented a q-deformation of this 3-algebra and noted it carrying the structure of ternary Hom–Nambu–Lie algebra. We know that the deep insights into the q-deformed algebra have been achieved. However for the q-deformed infinite-dimensional 3-algebra, much less is still known about its structure and property. As to the q-deformed infinite-dimensional n-algebra, to our best knowl-