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Vibrational quantisation of the $B = 7$ Skyrmion

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Abstract

We consider the inclusion of the most important vibrational modes in the quantisation of the dodecahedral $B = 7$ Skyrmion. In contrast to a rigid body quantisation, this formalism allows a spin $\frac{3}{2}$ state to lie below the spin $\frac{7}{2}$ state, in agreement with experimental data. There is also a low lying spin $\frac{1}{2}$ state and two spin $\frac{5}{2}$ states. We find that the excited spin $\frac{7}{2}$ state has a smaller root mean square charge radius than the other states. This prediction is an important signature of the Skyrme model, in conflict with more conventional nuclear models.

1. Introduction

The Skyrme model is a non-linear field theory of pions which admits soliton solutions called Skyrmions [1]. These are classically stable due to the topology of the system and each Skyrmion has a conserved topological charge, B. After quantisation Skyrmions are identified as nuclei with topological charge equal to baryon number.

The theory is non-renormalisable and so a first principles quantisation is beyond current methods. Instead, one must reduce the degrees of freedom in the problem to a finite number and quantise these. Each charge B Skyrmion may be separated into B charge one Skyrmions. These have six zero modes, three rotations and three translations. Thus to calculate quantities such as the binding energy of a nucleus one should take account of at least $6B$ degrees of freedom.

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Unfortunately this means quantising on a $6B$ dimensional space and little progress has been made, even for $B = 2$ [2]. Instead, one must select a subset of modes.

The simplest idea is to only include the zero modes of the Skyrmion, those transformations which leave the static energy unchanged. These are the rotations and isorotations (we stay in the centre of mass frame, allowing us to ignore translations). This procedure ignores vibrational modes, dynamical oscillations around the Skyrmion. Zero mode quantisation has had some success, such as reproducing the energy spectra of some light nuclei [3] and a natural description of the Hoyle state [4]. However, there are also some failures. For example, the binding energies are all much too large. This is to be expected when we truncate the degrees of freedom from $6B$ to 6.

Another failure of zero mode quantisation is the prediction of a spin $\frac{7}{2}$ ground state for the ^{7}Be/^{7}Li isodoublet. The dodecahedral symmetry of the $B = 7$ Skyrmion rules out low energy states with spin $\frac{1}{2}$, $\frac{3}{2}$ and $\frac{5}{2}$. In reality, experimental data show that all these states exist and the ground state has spin $\frac{3}{2}$. The first excited state of ^{7}Li has spin $\frac{1}{2}$ and lies 0.5 MeV above the ground state while the spin $\frac{7}{2}$ state is the second excited state lying 4.6 MeV above. In this paper we shall see that the inclusion of vibrational modes in the quantisation procedure resolves this problem.

The ^{7}Li and ^{7}Be nuclei are special. Among all nuclei with $B < 30$ they are the only ones that have an observed spin $\frac{7}{2}$ state lying below the lowest spin $\frac{5}{2}$ state. The $B = 7$ Skyrmion is also special. It has the largest finite symmetry group of any known Skyrmion with non-zero pion mass. We shall see that this large symmetry group is the reason why the spin $\frac{7}{2}$ state has abnormally low energy.

The ^{7}Li nucleus is usually described using a cluster model [5] which asserts that the nucleus is made of two interacting clusters. These are an alpha particle and a tritium nucleus. This model successfully reproduces the energy spectrum and some electrostatic properties of the nucleus. We shall see that the inclusion of vibrational modes in Skyrmion quantisation highlights a connection between the Skyrme model and the ideas of clustering.

This paper is organised as follows. In section 2 we review the Skyrme model and the structure of the $B = 7$ vibrational space. We discuss how one should include vibrations in the quantisation procedure and the effects of the Finkelstein–Rubinstein constraints in section 3. Details of the quantisation are laid out in section 4, alongside the results of our calculations and a comparison with the experimental data.

2. The $B = 7$ Skyrmion and its vibrational space

2.1. The Skyrme model

The Skyrme model can be defined in terms of the three pion fields, $\boldsymbol{\pi}(t, \boldsymbol{x})$. These are combined into an $SU(2)$-valued field

$$U(t, \boldsymbol{x}) = \sigma(t, \boldsymbol{x}) + i\boldsymbol{\pi}(t, \boldsymbol{x}) \cdot \boldsymbol{\tau}\,, \tag{2.1}$$

where $\boldsymbol{\tau}$ are the Pauli matrices and σ is an auxiliary field which satisfies $\sigma^2 + \boldsymbol{\pi} \cdot \boldsymbol{\pi} = 1$. This ensures that $U \in SU(2)$. Many quantities are most easily expressed in terms of the right current $R_\mu = (\partial_\mu U)U^\dagger$. The Lagrange density is given by

$$\mathcal{L} = -\frac{F_\pi^2}{16}\mathrm{Tr}\left(R_\mu R^\mu\right) + \frac{1}{32e^2}\mathrm{Tr}\left([R_\mu, R_\nu][R^\mu, R^\nu]\right) + \frac{1}{8}m_\pi^2 F_\pi^2\, \mathrm{Tr}(U - \mathbf{1}_2) \tag{2.2}$$