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Nuclear Physics B 904 (2016) 176–215

NUCLEAR PHYSICS B

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$\mathcal{N}=4$ superconformal Ward identities for correlation functions

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Received 22 September 2015; accepted 4 January 2016

Available online 7 January 2016

Editor: Stephan Stieberger

Abstract

In this paper we study the four-point correlation function of the energy–momentum supermultiplet in theories with $\mathcal{N}=4$ superconformal symmetry in four dimensions. We present a compact form of all component correlators as an invariant of a particular abelian subalgebra of the $\mathcal{N}=4$ superconformal algebra. This invariant is unique up to a single function of the conformal cross-ratios which is fixed by comparison with the correlation function of the lowest half-BPS scalar operators. Our analysis is independent of the dynamics of a specific theory, in particular it is valid in $\mathcal{N}=4$ super Yang–Mills theory for any value of the coupling constant. We discuss in great detail a subclass of component correlators, which is a crucial ingredient for the recent study of charge-flow correlations in conformal field theories. We compute the latter explicitly and elucidate the origin of the interesting relations among different types of flow correlations previously observed in arXiv:1309.1424.

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http://dx.doi.org/10.1016/j.nuclphysb.2016.01.008

1. Introduction

In this paper we study four-point correlation functions involving conserved currents in four-dimensional theories with $\mathcal{N} = 4$ superconformal symmetry. They include the R-symmetry current J_μ, the supersymmetry currents $(\Psi^\alpha_\mu, \bar{\Psi}^{\dot\alpha}_\mu)$ and the energy–momentum tensor $T_{\mu\nu}$. These operators belong to the so-called $\mathcal{N} = 4$ energy–momentum supermultiplet [1] and appear as various components in the expansion of the superfield $\mathcal{T}$ in powers of the 8 chiral (θ^A_α) and 8 antichiral $(\bar\theta^{\dot\alpha}_A)$ Grassmann variables, schematically,

$$\mathcal{T} = O(x) + (\theta\sigma^\mu\bar\theta)J_\mu(x) \\ + (\theta\sigma^\mu\bar\theta)\left[\theta\Psi_\mu(x) + \bar\theta\Psi_\mu(x)\right] + (\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta)T_{\mu\nu}(x) + \dots . \tag{1.1}$$

Here the lowest component is a half-BPS scalar operator O of dimension two, belonging to the representation $\mathbf{20}'$ of the R-symmetry group $SU(4)$. The superfield (1.1) satisfies a half-BPS 'shortening' condition, i.e., it is annihilated by half of the super-Poincaré generators. As a consequence, the expansion (1.1) is shorter than one would expect since $\mathcal{T}$ effectively depends on 4 chiral and 4 antichiral Grassmann variables only [2].

The central object of our study is the four-point correlation function of the energy–momentum supermultiplet (1.1) in $\mathcal{N} = 4$ superconformal theories. The most widely studied example is $\mathcal{N} = 4$ super-Yang–Mills theory (SYM) but in what follows we do not need to know any details about the dynamics of the theory. Our analysis is based solely on $\mathcal{N} = 4$ superconformal invariance and can be easily adapted to maximally supersymmetric theories in other space–time dimensions.

$\mathcal{N} = 4$ superconformal symmetry is powerful enough to fix the form of the two- and three-point correlation functions of $\mathcal{T}$'s [3–5]. In a perturbative theory, like $\mathcal{N} = 4$ SYM, the latter are protected from quantum corrections and only receive contributions at Born level [6]. The four-point correlation function (we use the notation $(i) \equiv (x_i, \theta_i, \bar\theta_i)$)

$$\mathcal{G}_4 = \langle \mathcal{T}(1) \dots \mathcal{T}(4)\rangle \tag{1.2}$$

is the first and simplest example of an unprotected quantity. It is this object that we study in the present paper.

The super-correlation function (1.2) combines together the correlation functions of various components of the multiplet (1.1). The latter appear as coefficients in the expansion of $\mathcal{G}_4$ in the Grassmann variables. The lowest component of $\mathcal{G}_4$ (with $\theta_i = \bar\theta_i = 0$) is the four-point correlation function of the half-BPS operators

$$\mathcal{G}_4\big|_{\theta_i=\bar\theta_i=0} = \langle O(x_1) \dots O(x_4)\rangle . \tag{1.3}$$

$\mathcal{N} = 4$ superconformal symmetry fixes this correlation function up to a single function $\Phi(u, v)$ of the two conformal cross-ratios u and v [7–9]. In the special case of $\mathcal{N} = 4$ SYM, this function comprises the dependence on the coupling constant. At weak coupling, its expansion in terms of scalar conformal integrals has been worked out up to six loops [10,11] and explicit expressions are available up to three-loop order [12–15]. At strong coupling, it has been computed within the AdS/CFT correspondence in the supergravity approximation [16–18].

A unique feature of the super-correlation function (1.2) is that the total number of Grassmann variables it depends upon (16 chiral and 16 antichiral variables) matches the total number of $\mathcal{N} = 4$ supercharges (Q^A_α, $\bar{Q}^{\dot\alpha}_A$, S^A_α and $\bar{S}^{\dot\alpha}_A$). As we show below, this property alone ensures that $\mathcal{N} = 4$ superconformal symmetry completely fixes all of its components, given the lowest one