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Massless conformal fields, $AdS_{(d+1)}/CFT_d$ higher spin algebras and their deformations

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Abstract

We extend our earlier work on the minimal unitary representation of $SO(d,2)$ and its deformations for $d = 4, 5$ and 6 to arbitrary dimensions d. We show that there is a one-to-one correspondence between the minrep of $SO(d,2)$ and its deformations and massless conformal fields in Minkowskian spacetimes in d dimensions. The minrep describes a massless conformal scalar field, and its deformations describe massless conformal fields of higher spin. The generators of Joseph ideal vanish identically as operators for the quasiconformal realization of the minrep, and its enveloping algebra yields directly the standard bosonic $AdS_{(d+1)}/CFT_d$ higher spin algebra. For deformed minreps the generators of certain deformations of Joseph ideal vanish as operators and their enveloping algebras lead to deformations of the standard bosonic higher spin algebra. In odd dimensions there is a unique deformation of the higher spin algebra corresponding to the spinor singleton. In even dimensions one finds infinitely many deformations of the higher spin algebra labelled by the eigenvalues of Casimir operator of the little group $SO(d-2)$ for massless representations.

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1. Introduction

In earlier work we studied the minimal unitary representation (minrep) of $SO(d, 2)$ for $d = 4, 5$ and 6 and their deformations using the quasiconformal approach. More specifically, in [1] we constructed the minrep of $SU(2, 2)$ and its one-parameter family of deformations which describe all massless conformal fields in four dimensions with the identification of the deformation parameter as helicity. The minrep of the superalgebras $\mathfrak{su}(2, 2|N)$ that extend the conformal algebra in four dimensions were also constructed in [1]. The minrep of $SU(2, 2|N)$ also admits an infinite family of deformation which describe massless N-extended superconformal multiplets in four dimensions. The minimal unitary supermultiplet of the $N = 4$ superconformal algebra $PSU(2, 2|4)$ is simply the $N = 4$ Yang–Mills supermultiplet.

These results were later extended to the construction of the minimal unitary representation of the six-dimensional conformal group $SO(6, 2) \approx SO^*(8)$ and its deformations and their supersymmetric extensions [2,3]. There exists a discrete infinite family of deformations of the minrep of $SO(6, 2)$, labelled by the eigenvalues of an $SU(2)$ subgroup of the little group $SO(4)$ of massless particles in $6d$. This infinite family of deformations of the minrep of $SO^*(8)$ describe massless conformal fields in six dimensions. The minimal unitary supermultiplets of $6d$ conformal superalgebras $\mathfrak{osp}(8^*|2N)$ also admit a discrete infinite family of deformations which describe massless conformal supermultiplets in six space–time dimensions.

In more recent work we constructed the minimal unitary representation of $SO(5, 2)$ using quasiconformal methods and showed that it admits a single deformation [4]. The minrep of $SO(5, 2)$ and its deformation are the analogs of scalar and spinor singletons of the three-dimensional conformal group $SO(3, 2)$, which is isomorphic to $Sp(4, \mathbb{R})$, and hence we referred to them as such. The Lie algebra of $SO(5, 2)$ admits a unique supersymmetric extension, namely the exceptional Lie superalgebra $\mathfrak{f}(4)$ with the even subalgebra $\mathfrak{so}(5, 2) \oplus \mathfrak{su}(2)$. The minimal unitary supermultiplet of $F(4)$ consists of the spinor singleton together with two copies of the scalar singleton [4].

The minrep of $SU(2, 2|N)$ and its deformations turn out to be isomorphic to the doubleton supermultiplets for integer and half-integer values of helicity [1] that were constructed and studied using twistorial oscillators some time ago [5–7]. The minimal unitary supermultiplet corresponds to the unique CPT self-conjugate doubleton supermultiplet. The twistorial oscillator method was used to obtain, for the first time, the Kaluza–Klein spectrum of IIB supergravity over $AdS_5 \times S^5$ simply by the tensoring of the CPT self-conjugate doubleton supermultiplet of $PSU(2, 2|4)$ [5]. The CPT self-conjugate doubleton of $PSU(2, 2|4)$ itself decouples from the Kaluza–Klein spectrum as gauge modes. Its Poincaré limit is singular and its field theory lives on the boundary of AdS_5 as conformally invariant $N = 4$ super Yang–Mills theory as was first pointed out in [5].

The minrep of $OSp(8^*|2N)$ and its deformations turn out to be isomorphic to the doubleton supermultiplets constructed using the twistorial oscillators [8–10]. The Kaluza–Klein spectrum of eleven-dimensional supergravity over $AdS_7 \times S^4$ was similarly obtained by the tensoring of the CPT self-conjugate doubleton supermultiplet of $OSp(8^*|4)$ [8]. The CPT self-conjugate doubleton supermultiplet does not have a Poincaré limit and its field theory lives on the boundary of AdS_7 as a conformally invariant field theory as was first pointed out in [8].

The physical importance of the four-dimensional $N = 4$ Yang–Mills supermultiplet and of the six-dimensional $(2, 0)$ conformal supermultiplet became abundantly clear after the fundamental paper of Maldacena [11] who proposed the duality between IIB superstring theory over $AdS_5 \times S^5$ and $SU(\mathcal{N})$ $N = 4$ Yang–Mills theory and M-theory over $AdS_7 \times S^4$ and the interacting six-dimensional conformal theory of $(2, 0)$ multiplets of Witten [12]. From a mathematical