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Orthogonal and symplectic Yangians and Yang–Baxter R-operators

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Abstract

Yang–Baxter R operators symmetric with respect to the orthogonal and symplectic algebras are considered in an uniform way. Explicit forms for the spinorial and metaplectic R operators are obtained. L operators, obeying the RLL relation with the orthogonal or symplectic fundamental R matrix, are considered in the interesting cases, where their expansion in inverse powers of the spectral parameter is truncated. Unlike the case of special linear algebra symmetry the truncation results in additional conditions on the Lie algebra generators of which the L operators is built and which can be fulfilled in distinguished representations only. Further, generalized L operators, obeying the modified RLL relation with the fundamental R matrix replaced by the spinorial or metaplectic one, are considered in the particular case of linear dependence on the spectral parameter. It is shown how by fusion with respect to the spinorial or metaplectic representation these first order spinorial L operators reproduce the ordinary L operators with second order truncation.

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1. Introduction

Let $\mathcal{G}$ be a Lie algebra of a Lie group G and V_j be spaces of representations ρ_j of $\mathcal{G}$ and G. We consider the Yang–Baxter (YB) relations in the general form

$$R_{12}(u)R_{13}(u+v)R_{23}(v) = R_{23}(v)R_{13}(u+v)R_{12}(u) \in \mathrm{End}(V_1 \otimes V_2 \otimes V_3)\,, \tag{1.1}$$

where the operator R_{ij} acts nontrivially only in the spaces V_i and V_j and u, v are spectral parameters. It is well known that (1.1) is the basic relation in the treatment of integrable quantum systems and is considered as an analog of the Jacobi identities in the formulation of the related algebras [1–7].

A solution $R_{ij}(u)$ of the YB relation (1.1) is called symmetric with respect to the group G or the algebra $\mathcal{G}$ if the action of $R_{ij}(u)$ on $V_i \otimes V_j$ commutes with the action of the group G (or its Lie algebra $\mathcal{G}$) in the representation $\rho_i \otimes \rho_j$:

$$[\rho_i(g) \otimes \rho_j(g)\,,\ R_{ij}(u)] = 0 \quad (\forall g \in G) \qquad \Leftrightarrow$$
$$[\rho_i(A) \otimes 1_j + 1_i \otimes \rho_j(A)\,,\ R_{ij}(u)] = 0 \ \ (\forall A \in \mathcal{G}).$$

The present paper is concerned with the specific features of the YB relations and the involved R operators in the cases of symmetry with respect to orthogonal (*so*) and symplectic (*sp*) algebra actions. The less trivial representation theories in those algebras compared to the special linear ($s\ell$) ones imply more involved structures in the Yang–Baxter R operators. A distinguishing feature of *so* and *sp* algebras compared to the $s\ell$ ones is the presence of an invariant metric – the second rank tensor ε, determining the scalar product in the defining (fundamental) representation. It is symmetric, $\varepsilon^T = \varepsilon$, in the *so* case and anti-symmetric, $\varepsilon^T = -\varepsilon$, in the *sp* case. In particular this results in the analogy between the *so* and *sp* cases connected with the interchange of symmetrization with anti-symmetrization and gives us the possibility to treat both cases simultaneously.

The *so* (or *sp*) symmetric matrix $R_{ij}(u)$ obeying the Yang–Baxter (YB) relation (1.1), where $V_1 = V_2 = V_3$ are spaces of a defining (fundamental) representation, is not a linear function in the spectral parameter u as it is for the $s\ell$ symmetric fundamental R-matrix. The explicit form of the fundamental *so* (and *sp*) symmetric R-matrices were found first in [7,8,11].

The generic YB relation (1.1) specifies to the *RLL* relation if two of the three spaces $V_1 = V_2 = V_f$ carry the fundamental representation ρ_f while the third space $V_3 = V$ is the space of any representation ρ of $\mathcal{G}$. In this case the R-operator acting on the product $V_f \otimes V$ is called L-operator (or L matrix). For the *so* and *sp* cases the *RLL* version of the YB relation involving the fundamental R matrices [7,8,11] together with the L operators of the form

$$L(u) = u\mathbf{1} + \frac{1}{2}\rho_f(G^a_{\ b})\,\rho(G^b_{\ a})\,, \tag{1.2}$$

does not hold for an arbitrary representation ρ of the generators $G^b_{\ a}$. The spinor representation ρ_s of the orthogonal algebra with $\rho_s(G_{ab}) = M_{ab} = \frac{1}{4}[\gamma_a, \gamma_b]$, where γ_a are Dirac gamma-matrices, is a distinguished case, where the *RLL* relation is obeyed with L of first order in the spectral parameter u (see (1.2)). Also the spinorial R matrix $R_{ss}(u)$, intertwining two spinor representations ρ_s, is known [9,10]. This and other representations of the orthogonal algebra distinguished in this sense as well as the corresponding R operators (including spinorial R operator) have been recently considered and analyzed in detail in [14,15].

As we mentioned above we rely on the known similarity of the *so* and *sp* algebras and treat the related R operators in a uniform way. In Section 2 we recall the fundamental R matrices