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Supersymmetric analogue of BC_N type rational integrable models with polarized spin reversal operators

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Abstract

We derive the exact spectra as well as partition functions for a class of BC_N type of spin Calogero models, whose Hamiltonians are constructed by using supersymmetric analogues of polarized spin reversal operators (SAPSRO). The strong coupling limit of these spin Calogero models yields BC_N type of Polychronakos–Frahm (PF) spin chains with SAPSRO. By applying the freezing trick, we obtain an exact expression for the partition functions of such PF spin chains. We also derive a formula which expresses the partition function of any BC_N type of PF spin chain with SAPSRO in terms of partition functions of several A_K types of supersymmetric PF spin chains, where $K \leqslant N-1$. Subsequently we show that an extended boson–fermion duality relation is obeyed by the partition functions of the BC_N type of PF chains with SAPSRO. Some spectral properties of these spin chains, like level density distribution and nearest neighbor spacing distribution, are also studied.

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1. Introduction

Remarkable progress has been made in recent years in the computation of exact spectra, partition functions and correlation functions of one-dimensional quantum integrable spin chains with long-range interactions as well as their supersymmetric generalizations [1–24]. Exact solutions of this type of quantum spin chains with periodic and open boundary conditions have been found to be closely connected with diverse areas of physics and mathematics like condensed matter systems exhibiting generalized exclusion statistics [5,23–25], quantum Hall effect [26], quantum electric transport phenomena [27,28], calculation of higher loop effects in the spectra of trace operators of planar $\mathcal{N}=4$ super Yang–Mills theory [29–31], Dunkl operators related to various root systems [32,33], random matrix theory [34], and Yangian quantum groups [4,5,9,17,35–37]. Furthermore, it has been recently observed that exactly solvable spin chains with long-range interactions can be generated through some lattice discretizations of conformal field theories related to the 'infinite matrix product states' [38–41].

The study of quantum integrable spin chains with long-range interactions was pioneered by Haldane and Shastry, who derived the exact spectrum of a spin-$\frac{1}{2}$ chain with lattice sites equally spaced on a circle and spins interacting through pairwise exchange interactions inversely proportional to the square of their chord distances [1,2]. It has been found that, the exact ground state wave function of this su(2) symmetric Haldane–Shastry (HS) spin chain coincides with the $U \to \infty$ limit of Gutzwiller's variational wave function describing the ground state of the one-dimensional Hubbard model [42–44]. A close relation between the su(m) generalizations of this HS spin chain and the (trigonometric) Sutherland model has been established by using the 'freezing trick' [6,45], which we briefly describe in the following. In contrast to the case of HS spin chain where lattice sites are fixed at equidistant positions on a circle, the particles of the su(m) spin Sutherland model can move on a circle and they contain both coordinate as well as spin degrees of freedom. However, in the strong coupling limit, the coordinates of these particles decouple from their spins and 'freeze' at the minimum value of the scalar part of the potential. Furthermore, this minimum value of the scalar part of the potential yields the equally spaced lattice points of the HS spin chain. As a result, in the strong coupling limit, the dynamics of the decoupled spin degrees of freedom of the su(m) spin Sutherland model is governed by the Hamiltonian of the su(m) HS model. Application of this freezing trick to the su(m) spin (rational) Calogero model leads to another quantum integrable spin chain with long-range interaction [6], which is known in the literature as the su(m) Polychronakos or Polychronakos–Frahm (PF) spin chain. The sites of such rational PF spin chain are inhomogeneously spaced on a line and, in fact, they coincide with the zeros of the Hermite polynomial [7]. Indeed, the Hamiltonian of the su(m) PF spin chain is given by

$$\mathcal{H}_{\rm PF}^{(m)} = \sum_{1\leqslant i<j\leqslant N} \frac{1-\epsilon P_{ij}^{(m)}}{(\rho_i-\rho_j)^2}\,, \tag{1.1}$$

where $\epsilon = 1$ (-1) corresponds to the ferromagnetic (anti-ferromagnetic) case, $P_{ij}^{(m)}$ denotes the exchange operator which interchanges the 'spins' (taking m possible values) of i-th and j-th lattice sites and ρ_i denotes the i-th zero of the Hermite polynomial of degree N. Due to the decoupling of the spin and coordinate degrees of freedom of the su(m) spin Calogero model for large values of its coupling constant, an exact expression for the partition function of su(m) PF spin chain can be derived by dividing the partition function of the su(m) spin Calogero model through that of the spinless Calogero model [8]. Similarly, the partition function of su(m) HS